Introduction to PID Control

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Introduction

- Historical Review
- Functions of Control System
- Standard PID Structure
- Control Action of the PID Controller
- Characteristics of PID Control Action

PID – The Most Popular Controller in Practice

This talk examines a particular control structure that has become almost universally used in industrial control. It is based on a particular fixed structure controller family, the so-called PID controller family. These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for: **P** (*Proportional*)

I (*Integral*)

D (*Derivative*)

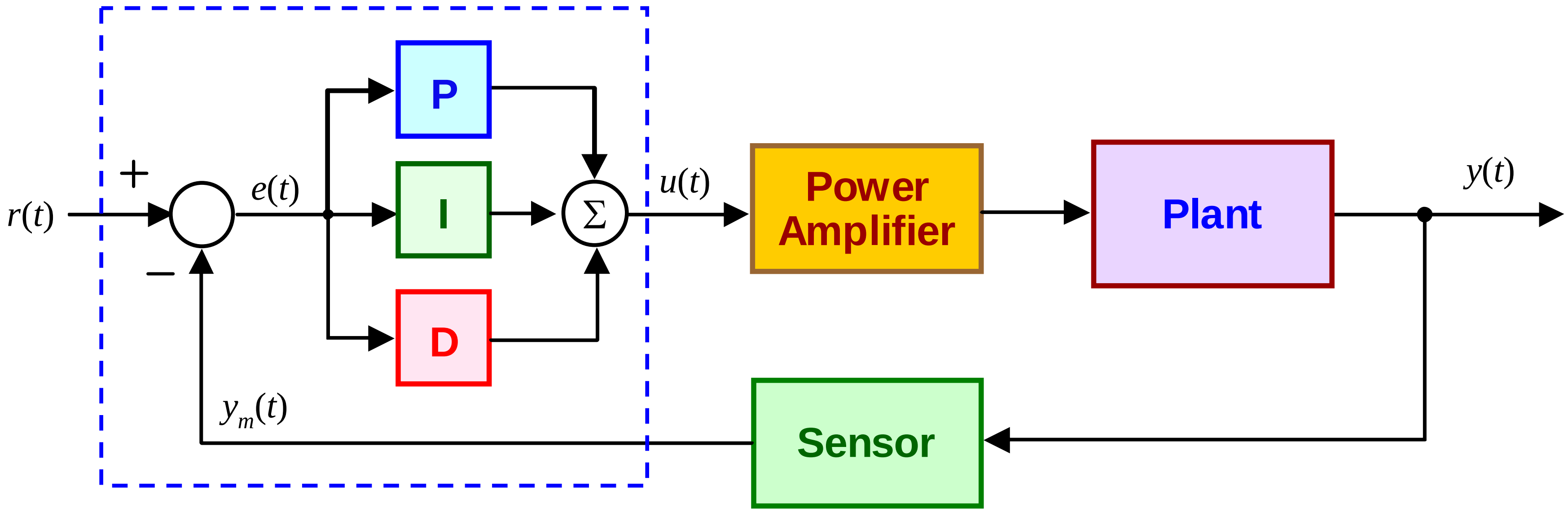
Historical Note

Early feedback control devices implicitly or explicitly used the ideas of proportional, integral and derivative action in their structures. However, it was probably not until Minorsky's work on ship steering* published in 1922, that rigorous theoretical consideration was given to PID control.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

Minorsky (1922) "Directional stability of automatically steered bodies", J. Am. Soc. Naval Eng., 34, pp. 284.

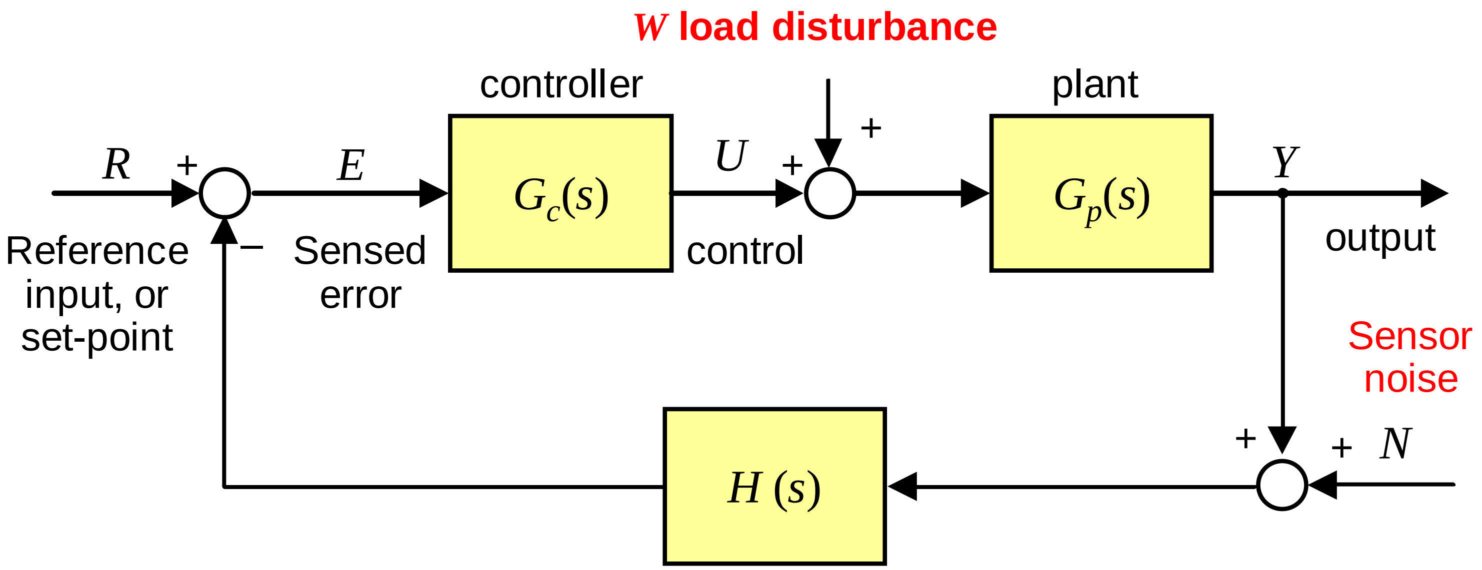
The Current Situation



Despite the abundance of sophisticated tools, including advanced controllers, the Proportional, Integral, Derivative (PID controller) is still the most widely used in modern industry, controlling more that 95% of closed-loop industrial processes*.

- Åström K.J. & Hägglund T.H. 1995, "New tuning methods for PID controllers", Proc. 3rd European Control Conference, pp. 2456-62; and
- Yamamoto & Hashimoto 1991, "Present status and future needs: The view from Japanese industry", Chemical Process Control, CPCIV, Proc. 4th Inter-national Conference on Chemical Process Control, Texas, pp.1-28.

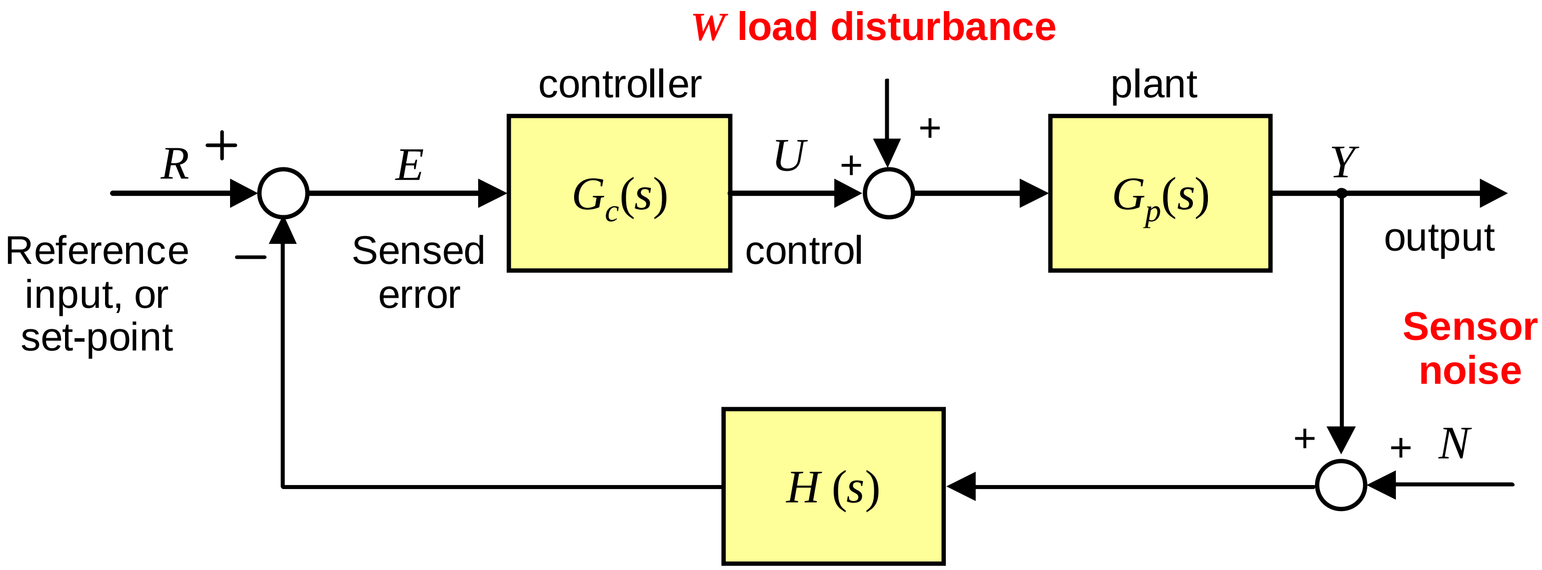
Functions of Control System



$$Y(s)=\frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)H(s)}R(s)+\frac{G_p(s)}{1+G_c(s)G_p(s)H(s)}N(s)+\frac{H(s)G_c(s)G_p(s)}{1+G_c(s)G_p(s)H(s)}W(s)$$

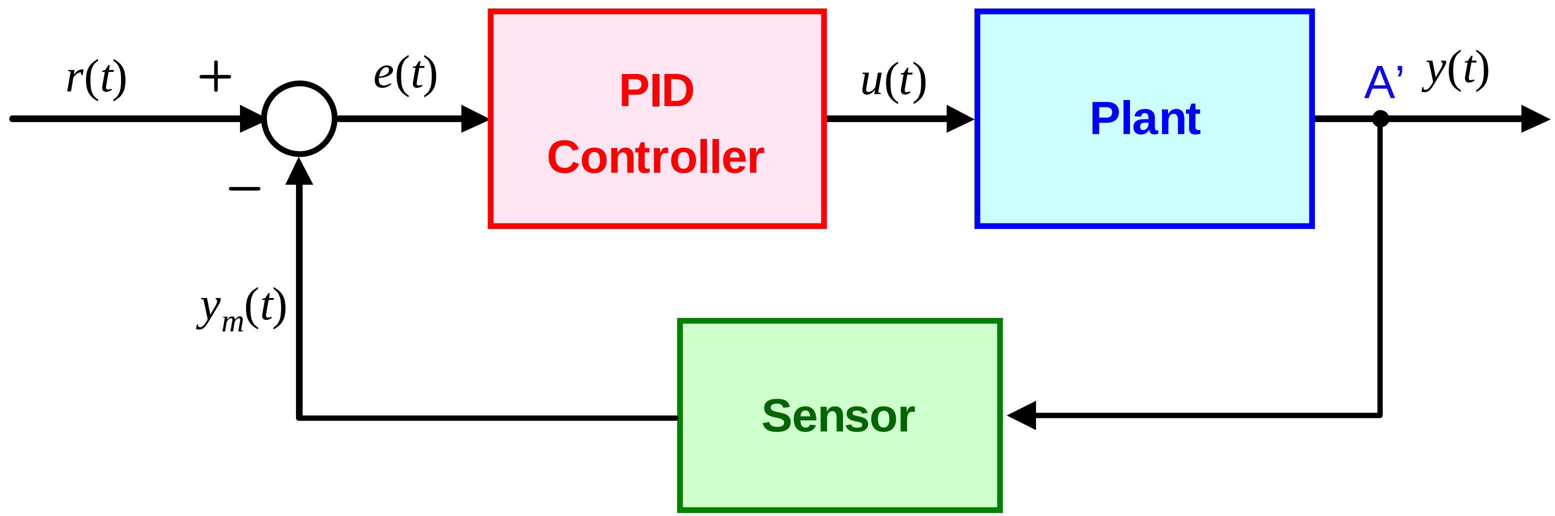
- Track reference input, or maintain set point despite:
  - Load disturbances (usually low frequency)
  - Sensor noise (usually high frequency)
- Achieve specified bandwidth, and transient response characteristics

Performance of Control System

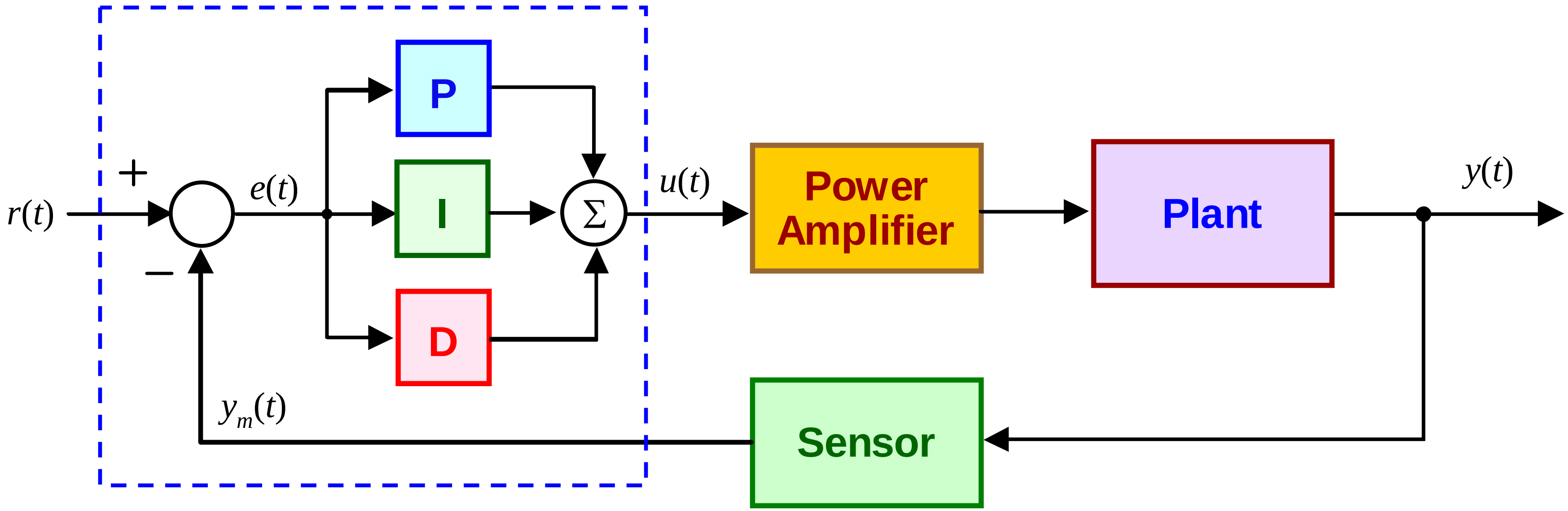


- Sensor noise reproduced just lick reference input
  - Use low noise sensors!
  - Seek to make $\dfrac{G_c G_p(s)}{1+G_c G_p(s)} \rightarrow \begin{cases} 1 \text{ at low freq.} \\ 0 \text{ at high freq.} \end{cases}$
- To reject disturbances, make $1+G_c G_p(s) \rightarrow \infty$ at disturbance freq.

Standard PID Structure



Control Action of the PID Controller



- The standard form PID controller generates its control action according to the error

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{d\,e(t)}{dt} \Rightarrow u(t) = K_P e(t) + \frac{1}{T_I}\int e(t)dt + T_D \frac{d\,e(t)}{dt}$$

Proportional + Integral + Derivative

Analog PID Controller Equations

The equation of the analog PID controller is

$$u(t) = K_P e(t) + K_I \int e(t) d\, t + K_D \frac{d\, e(t)}{d\, t}$$

The transfer function of the PID controller is $H_{\text{PID}}(s) = K_P + \frac{K_I}{s} + K_D\, s$

It can also be expressed as

$$H_{\text{PID}}(s) = K(1 + \frac{1}{T_I\, s}) + T_D\, s$$

in which K specifies the proportional gain, T_I characterizes the integral action and is called the integral time constant, and T_D characterizes the derivative action and is called the derivative time constant.

The integral term of a PID controller can eliminate steady-state error for a step reference.

Integral Time Constant and Derivative Time Constant

The integral term of the PID controller is

$$u(t) = K_I \int e(t)dt \;\Rightarrow\; u(t) = \frac{1}{T_I}\int e(t)dt \;\Rightarrow\; T_I = \frac{1}{K_I}$$

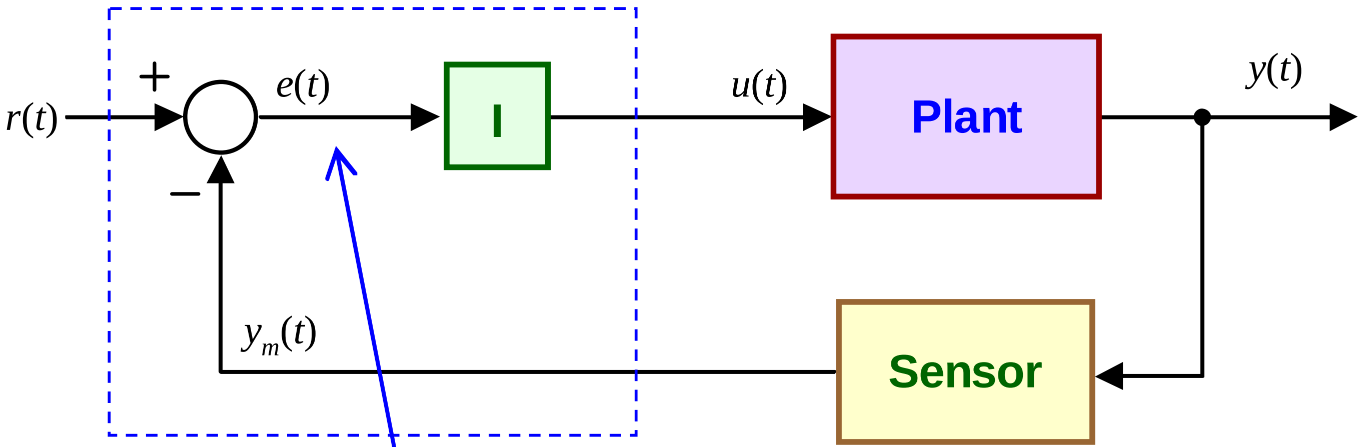
The integral time constant is defined as the period for which its integration effect of the error is equivalent to the proportional action.

The derivative term of the PID controller is

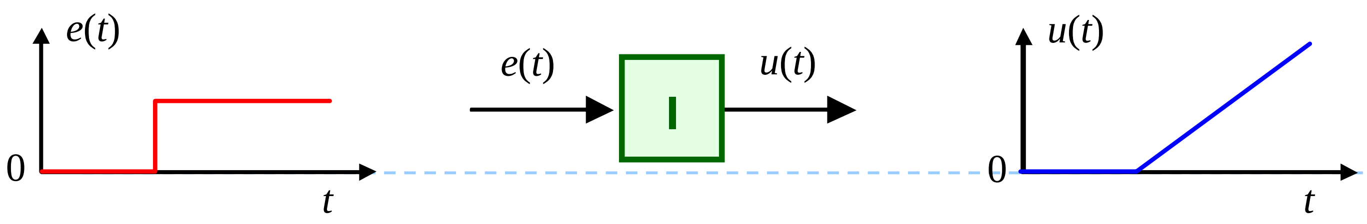
$$u(t) = K_D \frac{d\,e(t)}{d\,t} \;\Rightarrow\; u(t) = T_D \int e(t)dt \;\Rightarrow\; T_D = K_D$$

The differential time constant is defined as the period for which its differentiation effect of the error is equivalent to the proportional action.

Characteristics of Integral Control Action

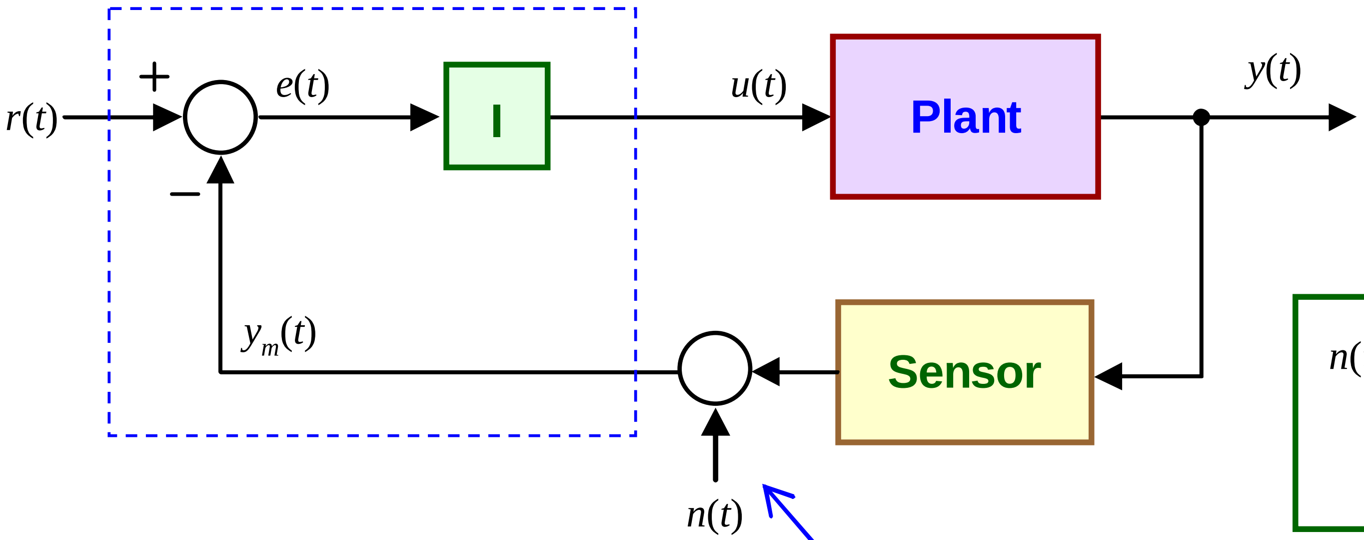


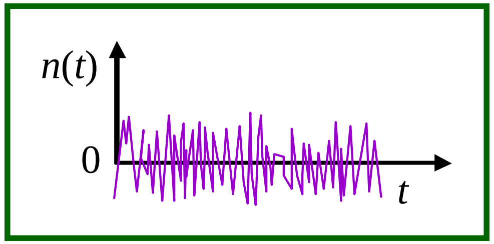
If there exists a constant error, the integrator will integrate this error to a ramping actuation signal.

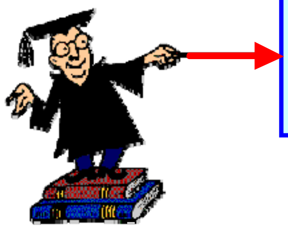


Zero steady-state error for a dc reference.

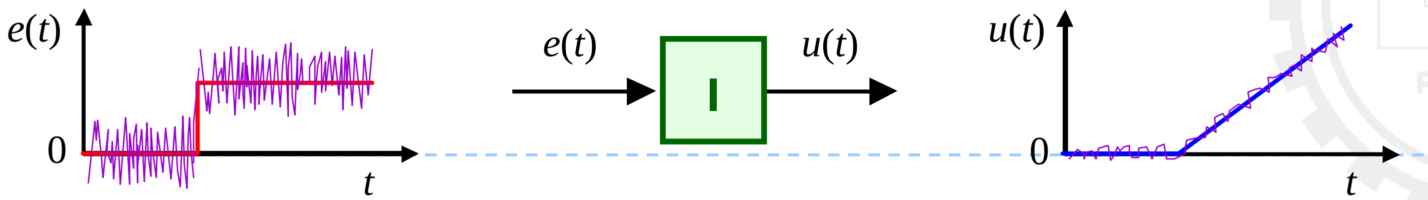
Characteristics of Integral Control Action



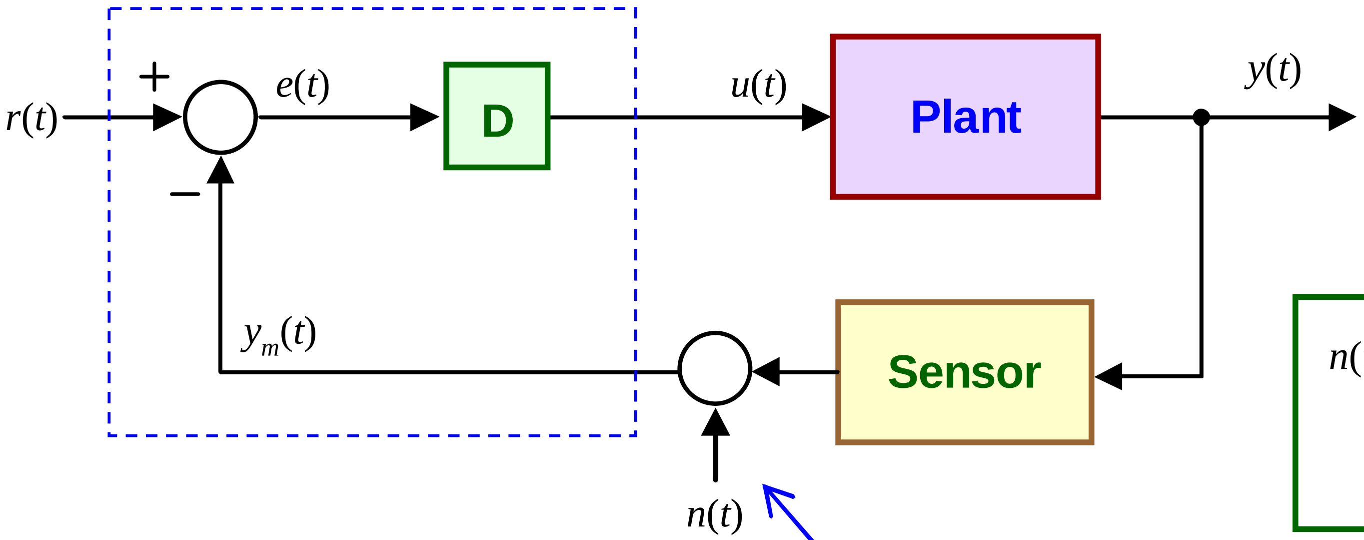


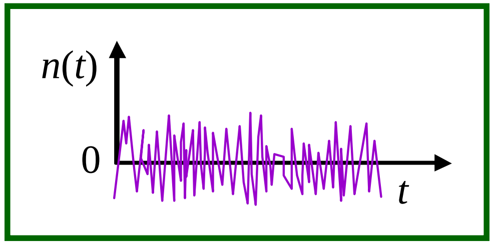


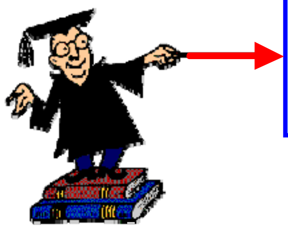
If there exists sensing noises, the integrator has an inherent smoothing effect!



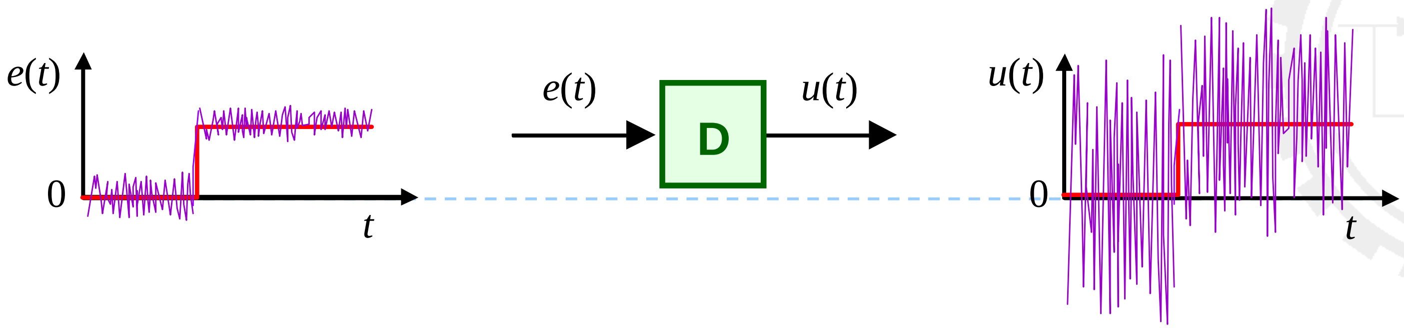
Characteristics of Derivative Control Action







If there exists sensing noises, the differentiator has an inherent amplification effect!



Standard PID Form

The *standard form* PID are:

Proportional: → $G_P(s) = K_P$

Proportional + Integral: → $G_{PID}(s) = K_P\left(1 + \frac{1}{T_i\, s}\right)$

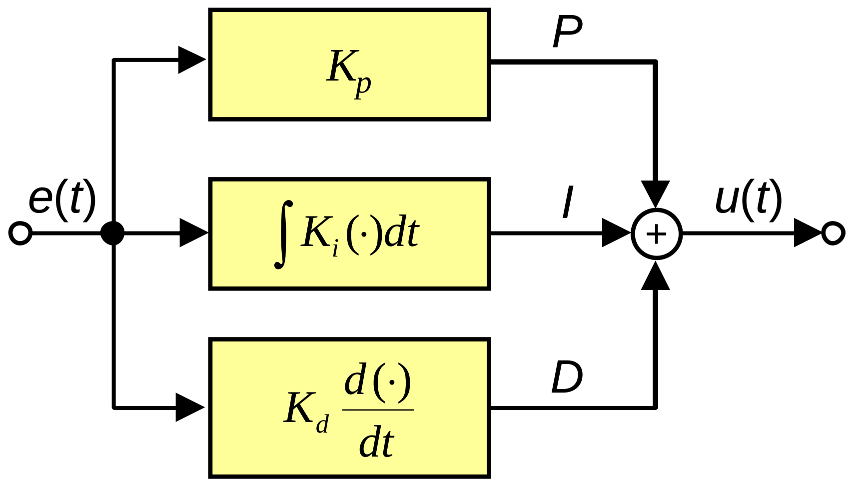
Proportional + Derivative: → $G_{PID}(s) = K_P\left(1 + \frac{T_d s}{\tau_D s + 1}\right)$

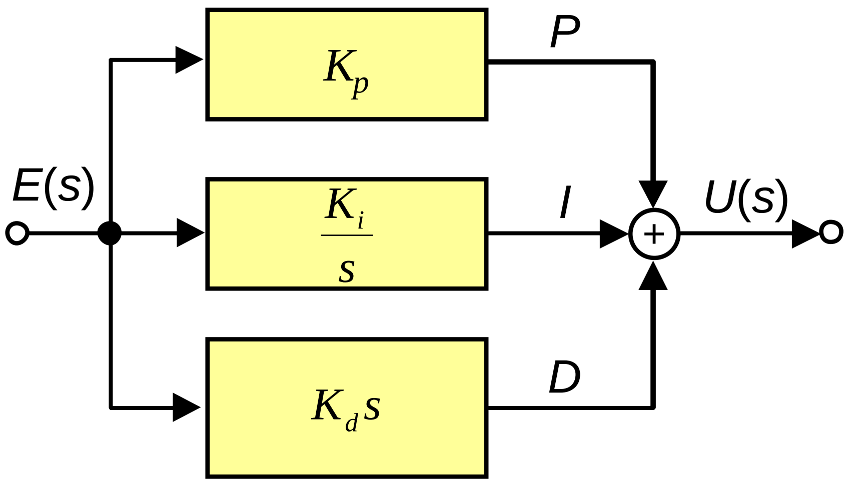
Proportional + Integral + Derivative: → $G_{PID}(s) = K_P\left(1 + \frac{1}{T_i\, s} + \frac{T_d s}{\tau_D s + 1}\right)$

Characteristics of PID Controllers

- **Provides set point regulation (error zeroing) against arbitrary disturbances (as long as they are low frequency)**
- **Is robust against modeling errors**
- **Is non-fragile in general**
- **Three term controllers are easier to adjust at the design stage as well as online**

PID Controller Functions





- Output feedback
  - From proportional action
- Eliminate steady-state offset
  - From integral action
- Anticipation
  - From derivative action

Compare output with set-point

Apply constant control even when error is zero

React to rapid rate of change Before error grows too big

Transfer Function of PID Controller

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$$

$$= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \quad \text{where } T_i = \frac{K_p}{K_i}, T_d = \frac{K_d}{K_p}$$

$$= K_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s}$$

Derivative time constant

Integral time constant, or "reset time"

- If no derivative action, we have PI controller:

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s}$$

$$= K_p\left(1 + \frac{1}{T_i s}\right) \quad \text{where } T_i = \frac{K_p}{K_i}$$

Proportional gain

Integral gain

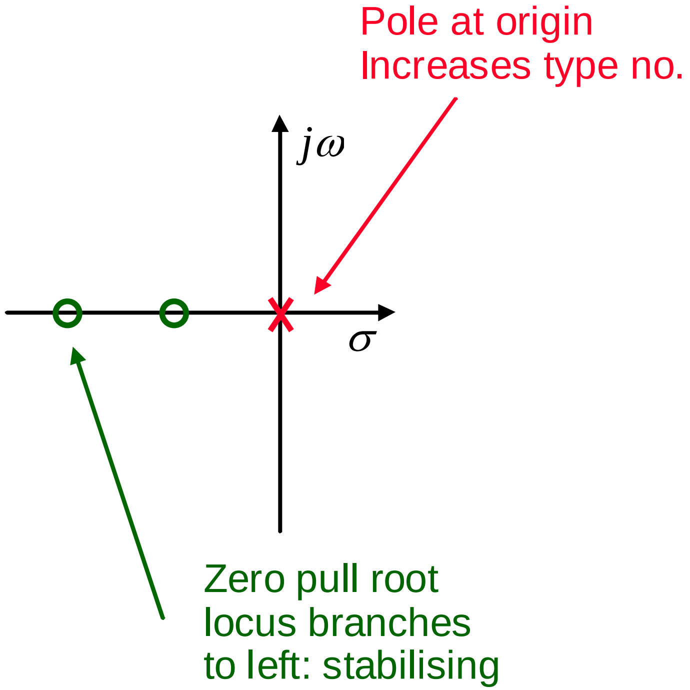
$$= K_p \frac{T_i s + 1}{T_i s}$$

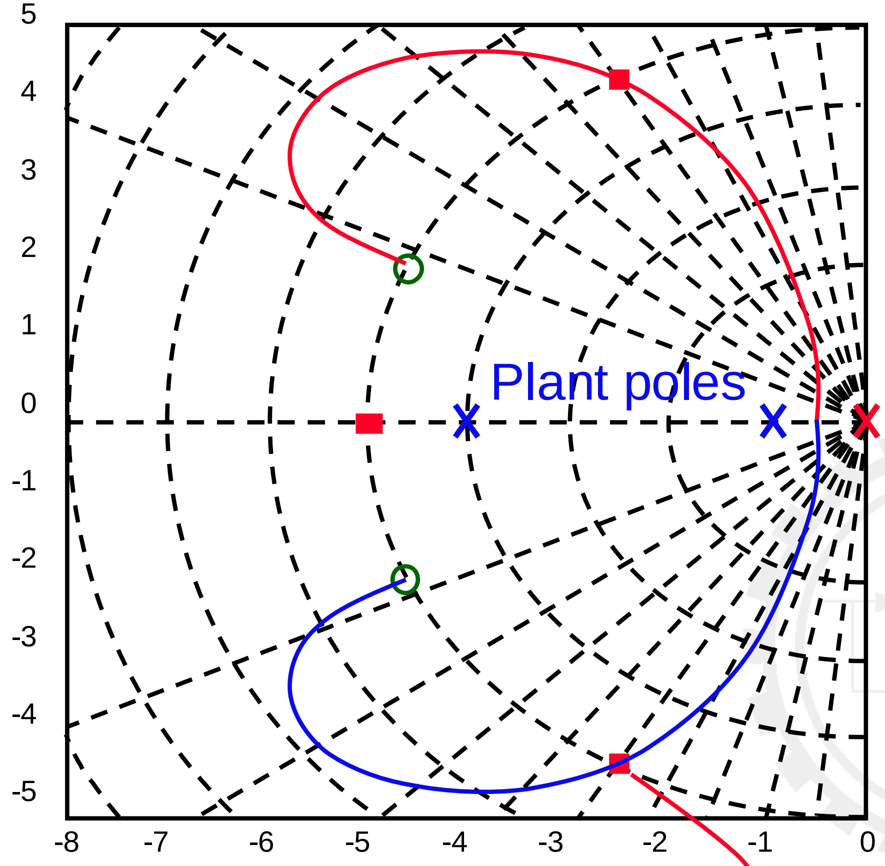
Effect on Open-Loop Transfer Function

$$G_c(s) = K_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s}$$

Example: $G_p = \dfrac{4}{(s+1)(s+4)}, \; T_i = 0.37, \; T_d = 0.11$

- S-plane





Closed-loop poles for kp = 11.5

Effect on Open-Loop Transfer Function

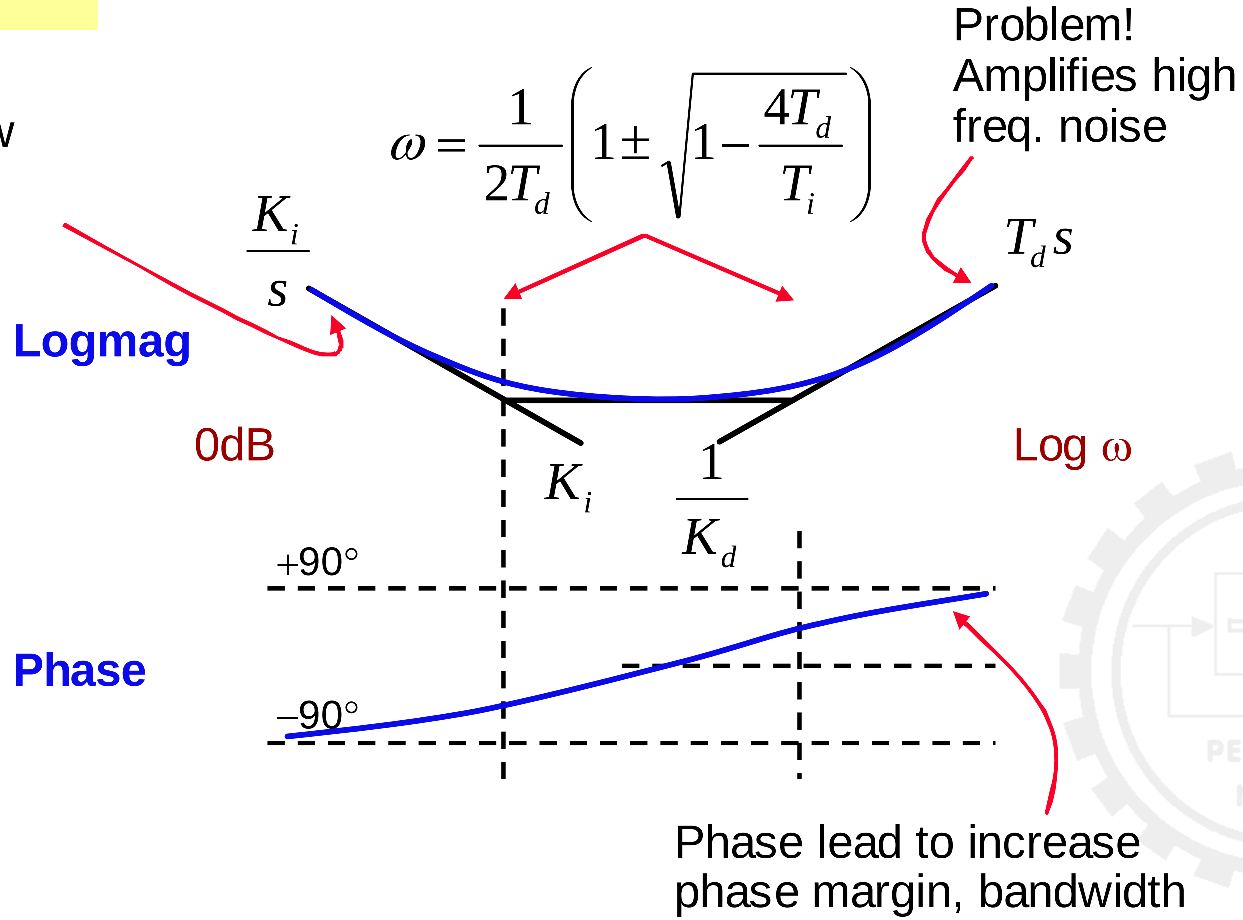
$$G_c(s) = K_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s}$$

- Frequency response

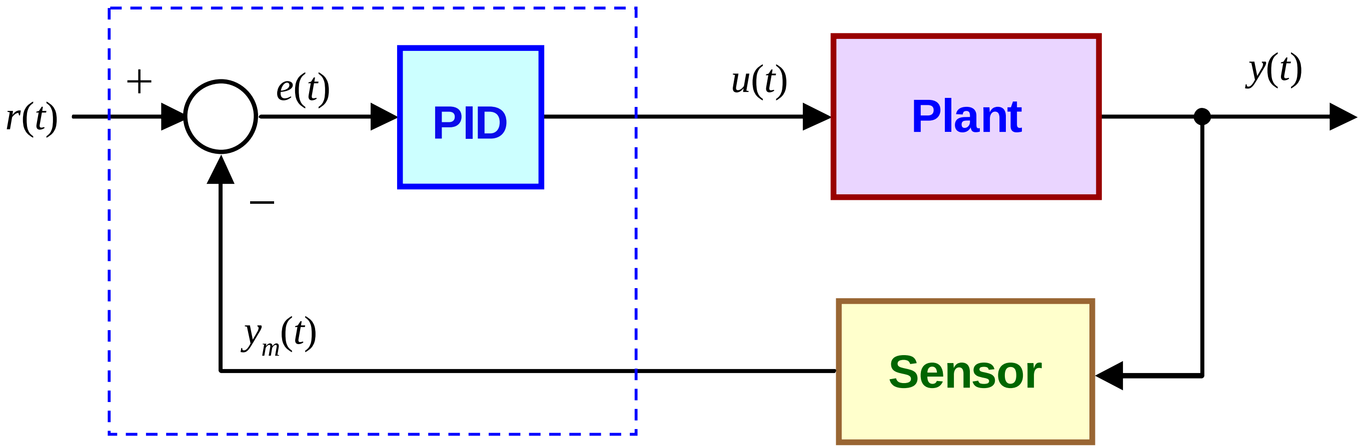
Amplitude boost at low frequencies to reduce steady-state error

$$G_c \rightarrow \infty \text{ as } s \rightarrow 0$$

$$\frac{G_c G_p}{1+G_c G_p} \rightarrow 1$$

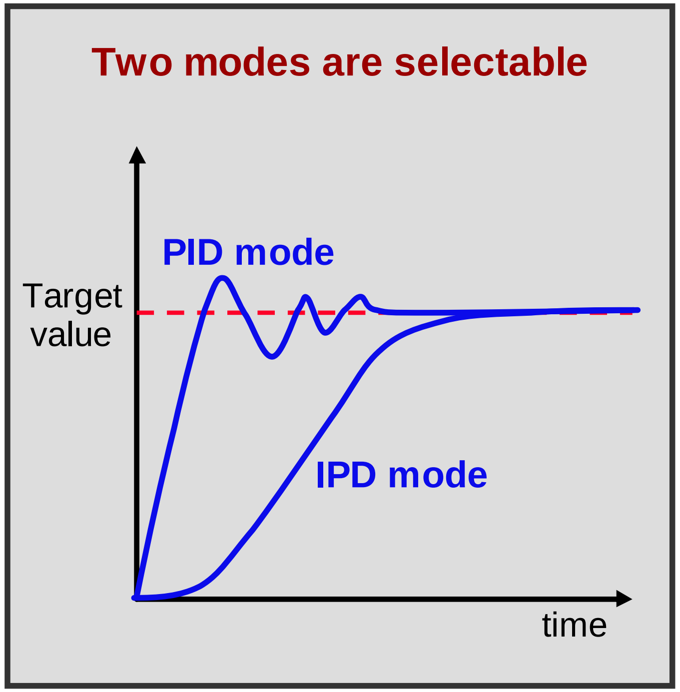


Physical Interpretation of the PID Control



	Rise time	Maximum overshoot	Settling time	Steady-state error
P	**Decrease**	Increase	Small change	**Decrease**
I	**Decrease**	Increase	Increase	**Eliminate**
D	Small change	**Decrease**	**Decrease**	Small change

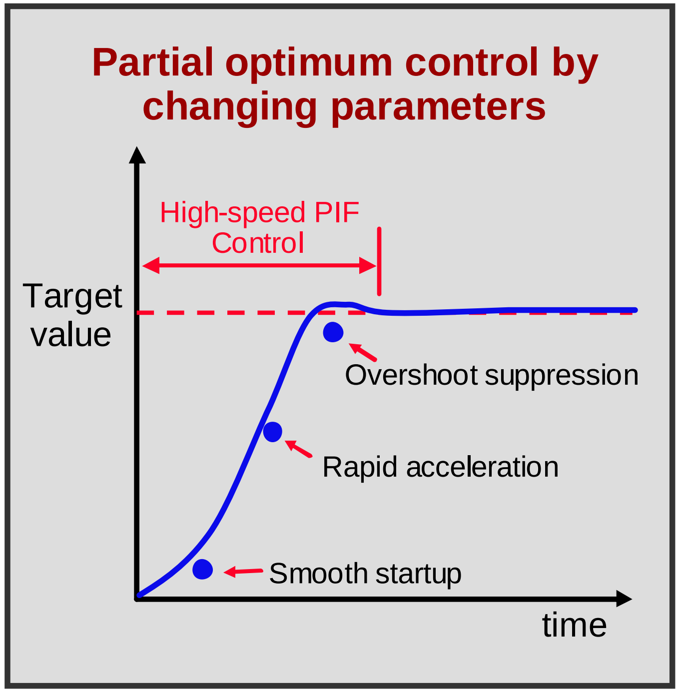
IPD vs. PID Control



From industry experiences, more than 90% of the time IPD works better than PID in applications of temperature control.

P. K. Nandam and P.C. Sen, "Analog and digital speed control of DC drives using proportional-integral and integral-proportional control techniques," *IEEE Trans. on Ind. Electron.*, vol. 34, no. 2, pp. 227-233, May 1987 .

Gain Scheduling PID Control

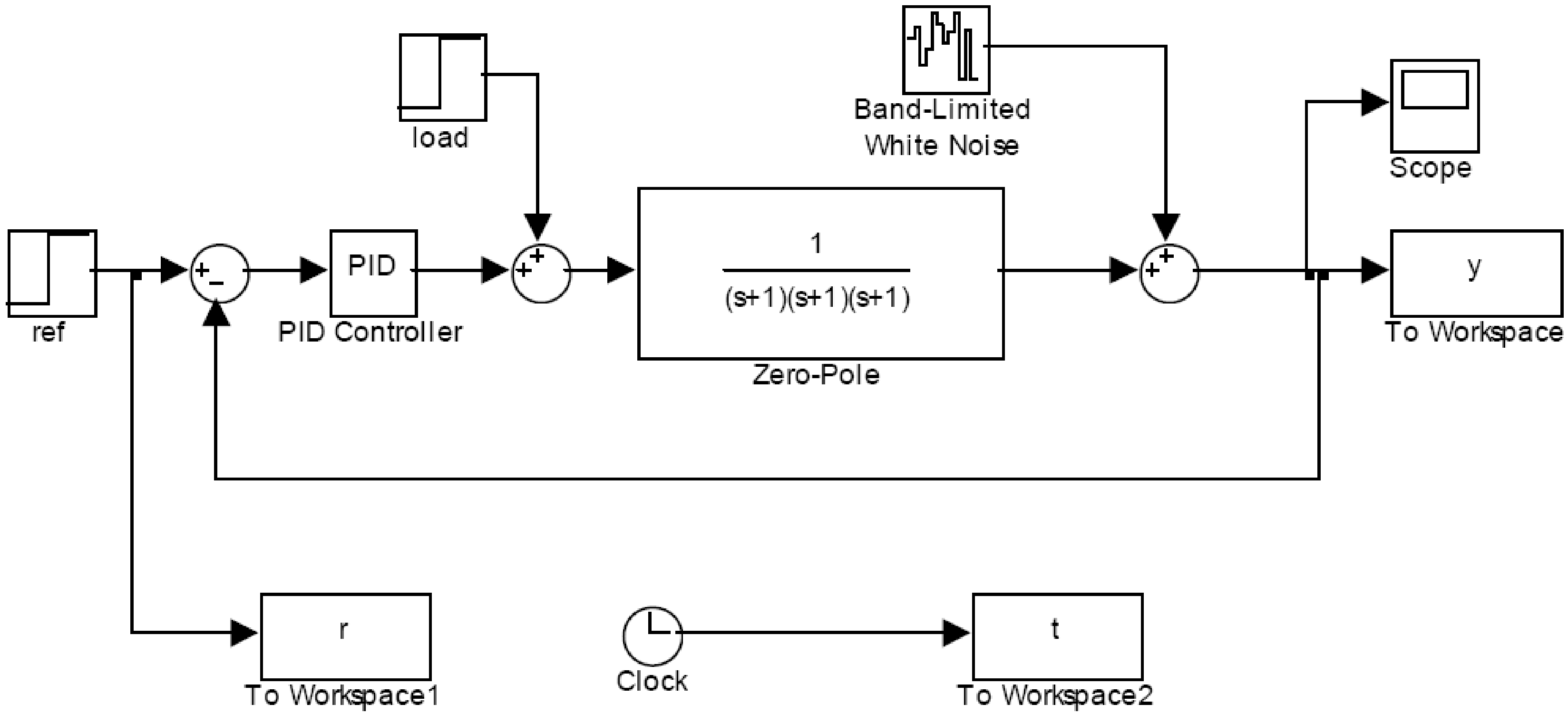


By combining with a sequence control logic, the parameters (Kp, Ti, Td, etc.) can be changed during a PID control execution, thereby enabling optimum temperature control in each stage including start up, mid-range, and convergence.

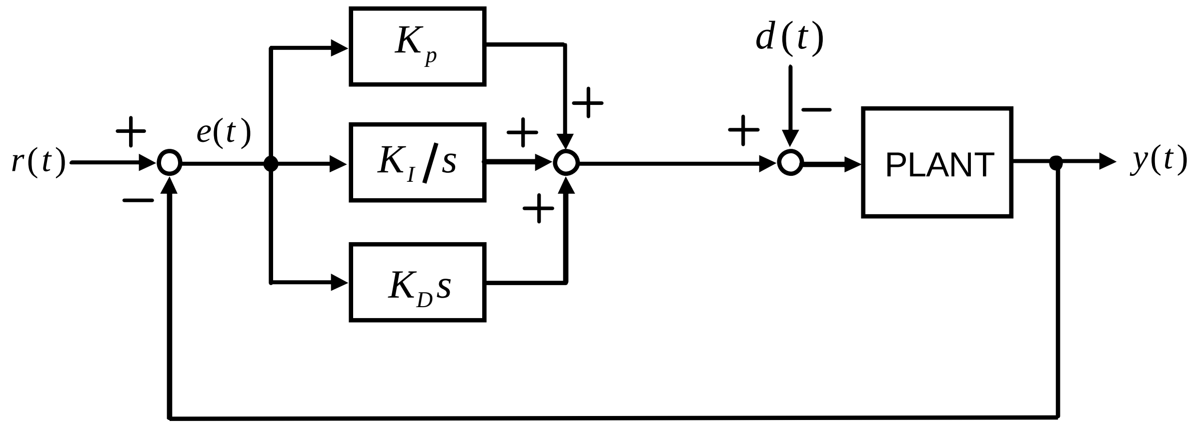
Application of PID Control

- **PID regulators provide reasonable control of most industrial process, provided performance demands not too high.**
- **PI control generally adequate when plant/process dynamics are essentially first-order**
  - **Plant operators often switch D-action off; due to “difficult to tune”**
- **PID control generally OK if dominant plant dynamics are 2nd-order**
- **More elaborate control strategies needed if process has logh time delay, or under lightly-damped vibratioon modes**

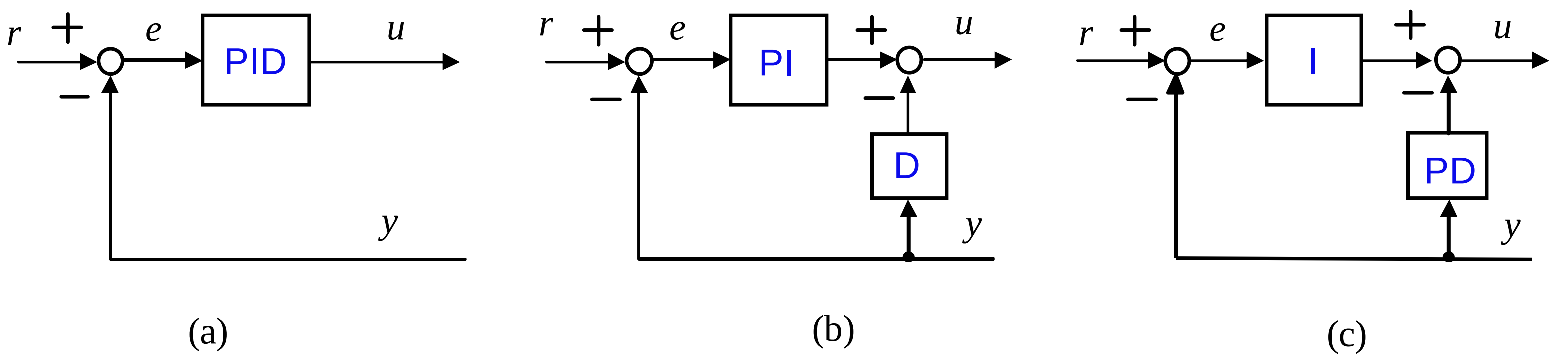
Simulink Model for PID Control



Derivatives of PID Controller

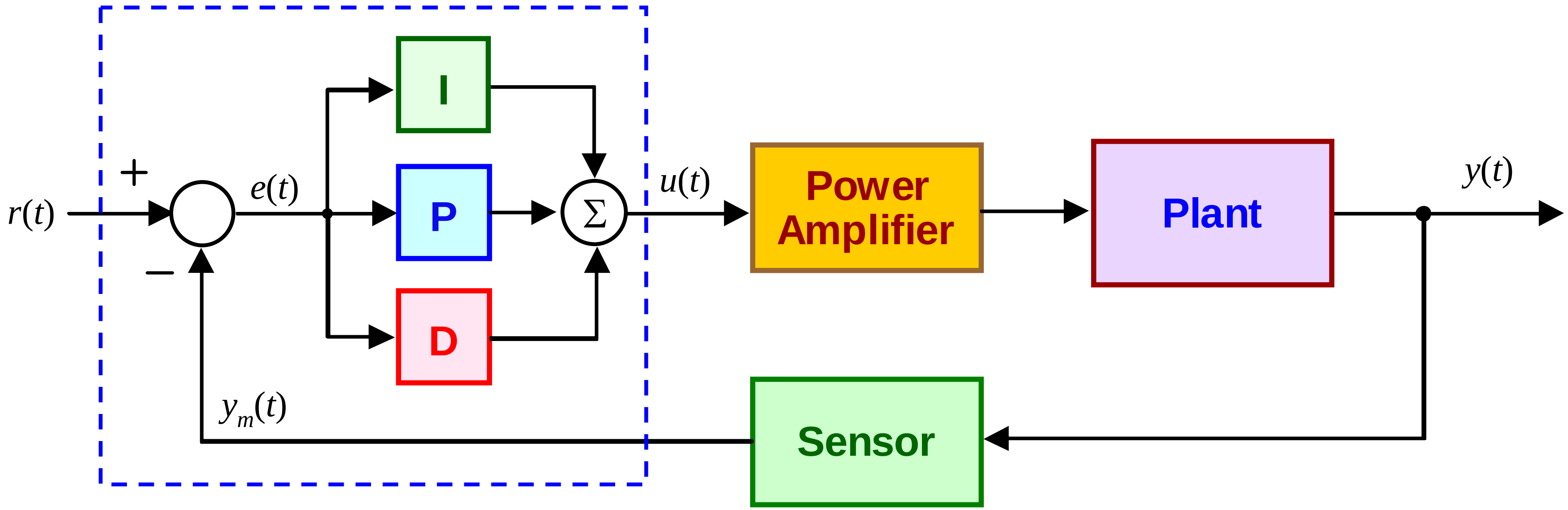


Analog PID Controller



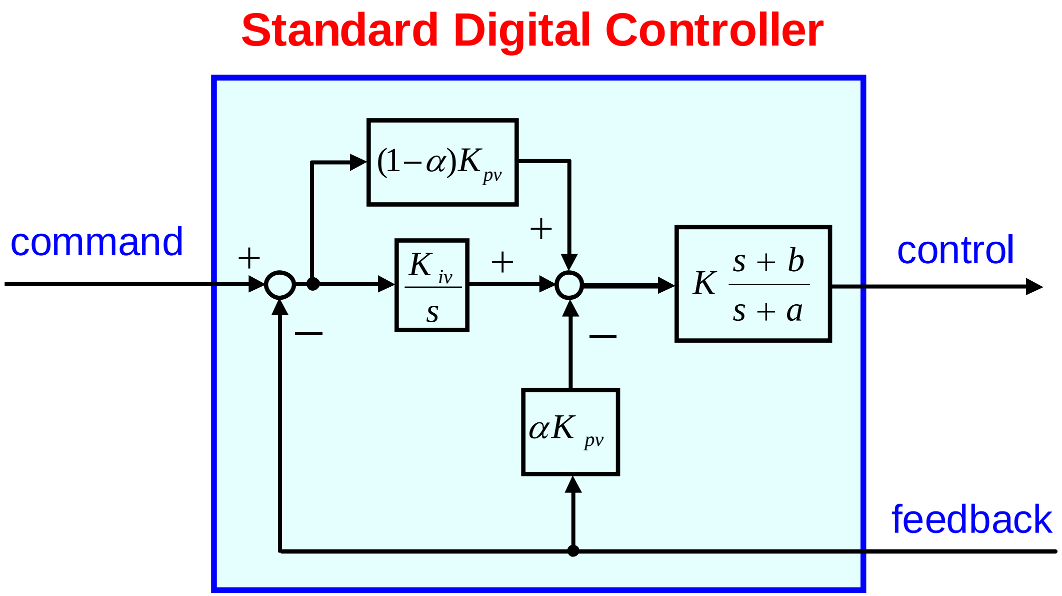
(a) Textbook PID controller, (b) derivative-of-output controller, (c) IPD controller.

Various Forms of PID Controller



How many forms we can derive from the PID control action?

Hybrid IP & PI Control with Phase-Lead Compensation



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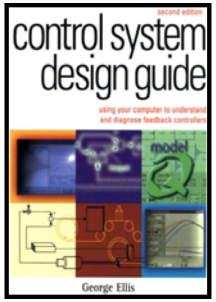
2. P. K. Nandam and P.C. Sen, "Analog and digital speed control of DC drives using proportional-integral and integral-proportional control techniques," *IEEE Trans. on Ind. Electron.*, vol. 34, no. 2, pp. 227-233, May 1987.
3. C. L. Phillips and J. M. Parr, "Robust design of a digital PID predictor controller," *IEEE Trans.on Ind. Electron.*, vol. 31, no. 4, pp. 328-332, Nov. 1984.

Recommend Books: PID Control



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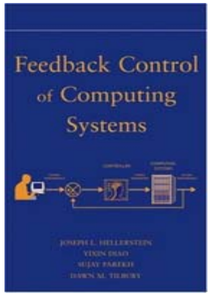

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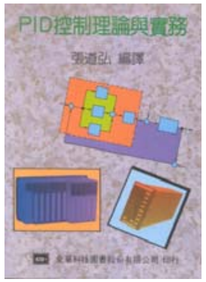

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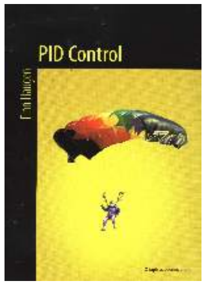

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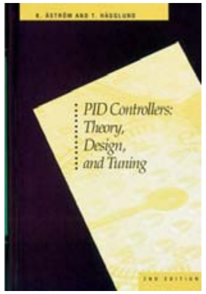
Finn Haugen, Tapir Academic Press, July 2004.

Recommend Books: PID Control



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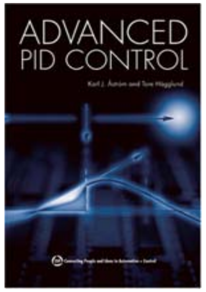
International Society of America, 2nd Ed., Jan. 1, 1995.



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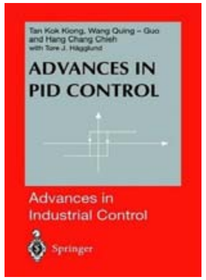
The Instrumentation, Systems, and Automation Society, August 15, 2005.



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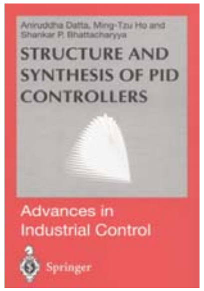

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Springer-Verlag; 1st edition, May 15, 2000.

Recommend Books: PID Control

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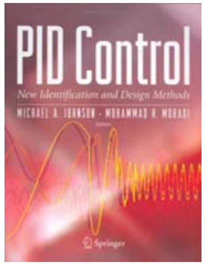

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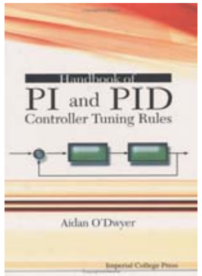
Michael A. Johnson (Editor) and Mohammad H. Moradi (Editor), Springer, May 26, 2005.



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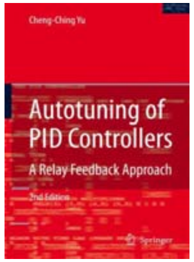

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