PWM Switch Model

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Power Electronic Systems & Chips, NCTU, TAIWAN
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PWM Switch Model

- Introduction
- PWM Switch and its Invariant Properties
- CCM Analysis
  - DC and Small-Signal Model of PWM Switch
  - PWM Switch Model of Buck Converters
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- DCM Analysis
  - DC and Small Model of PWM Switch
  - Analysis of PWM Converters
  - Zero of Control-to-Output Transfer Function in DCM

PWM Switch Model (1990)

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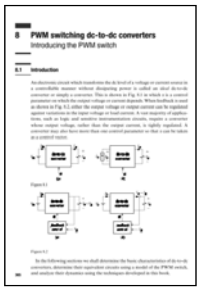
- **PWM Switch Method**

[1] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part I: **Continuous Conduction Mode**," *IEEE Trans. on Aero. and Elec. Sys.*, vol. 26, no. 3, pp. 490-496, May 1990.
[2] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part II: **Discontinuous Conduction Mode**," *IEEE Trans. on Aero. and Electron. Sys.*, vol. 26, no. 3, pp. 497-505, May 1990.

Fast Analytical Techniques for Electrical and Electronic Circuits,
V. Vorperian, Cambridge Press, 2004.



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Pulse-Width Modulator

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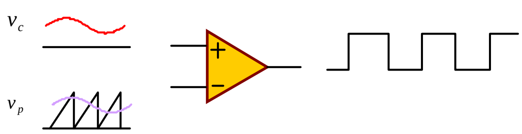
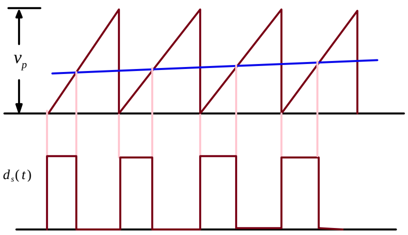


For natural sampling,



$$D = \frac{T_{ON}}{T_S} = \frac{v_c}{v_p}$$

$$\hat{d}(s) = K_m v_c(s)$$

$$K_m = \frac{1}{v_p} = \text{constant}$$

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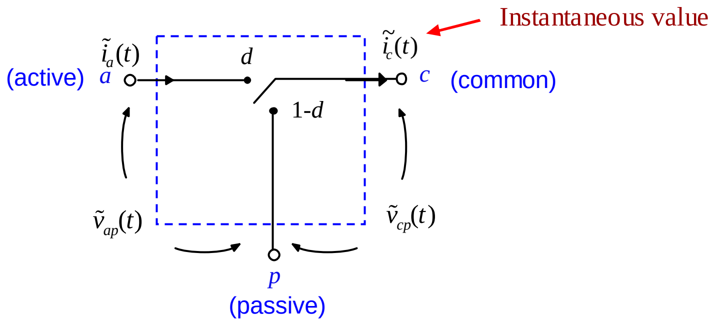
PWM Switch Model



PWM Switch Modeling

[1] V. Vorperian, R. Tymerski, and F.C.Y. Lee, "Equivalent circuit models for resonant and PWM switches," *IEEE Transactions on Power Electronics*, vol. 4, no. 2, pp. 205-214, Apr 1989.

[1] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part I: Continuous Conduction Mode," *IEEE Trans. on Aero. and Electron. Sys.*, vol. 26, no. 3, pp. 490-496, May 1990.

[2] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part II: Discontinuous Conduction Mode," *IEEE Trans. on Aero. and Electron. Sys.*, vol. 26, no. 3, pp. 497-505, May 1990.

[3] E. Van Dijk, J. N. Spruijt, D. M. O'Sullivan, and J. B. Klaassens, "PWM-switch modeling of DC-DC converters," *IEEE Transactions on Power Electronics*,, vol. 10, no. 6, pp. 659 -665, Nov 1995.

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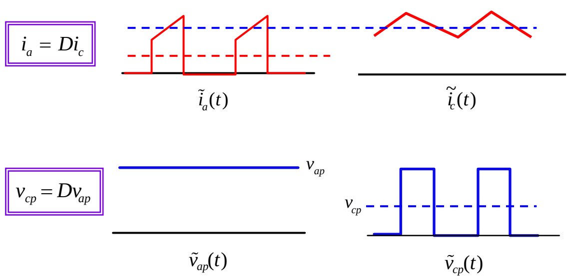
Modeling of the Switch



Note: i_a denotes the average value of $\tilde{i}_a(t)$

During a PWM switching interval:

$$\tilde{i}_a(t)=\begin{cases}\tilde{i}_c(t), & 0\le t\le DT_s\\ 0, & DT_s\le t\le T_s\end{cases}\qquad \tilde{v}_{cp}(t)=\begin{cases}\tilde{v}_{ap}(t), & 0\le t\le DT_s\\ 0, & DT_s\le t\le T_s\end{cases}$$

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Small-Signal Perturbation

Make a small-signal perturbation at a DC operating point

$$d = D + \hat{d} \qquad i_a = I_a + \hat{i}_a \qquad v_{cp} = V_{cp} + \hat{v}_{cp}$$
$$i_c = I_c + \hat{i}_c \qquad v_{ap} = V_{ap} + \hat{v}_{ap}$$

We can obtain

$$i_a = D i_c \;\Rightarrow\; \begin{aligned} i_a &= d i_c \\ I_a + \hat{i}_a &= (D + \hat{d})(I_c + \hat{i}_c) \\ I_a + \hat{i}_a &= D I_c + I_c \hat{d} + D\hat{i}_c + \hat{d}\hat{i}_c \end{aligned}$$

$$v_{cp} = D v_{ap} \;\Rightarrow\; \begin{aligned} v_{cp} &= d v_{ap} \\ V_{cp} + \hat{v}_{cp} &= (D + \hat{d})(V_{ap} + \hat{v}_{ap}) \\ V_{cp} + \hat{v}_{cp} &= D V_{ap} + V_{ap}\hat{d} + D\hat{v}_{ap} + \hat{d}\hat{v}_{ap} \end{aligned}$$

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DC and AC Analysis

For dc analysis, eliminate all small signal perturbation terms, we can get its equivalent dc model.

$$I_a = D I_c$$
$$V_{cp} = D V_{ap}$$

For ac analysis, set all dc terms to zero and neglect second order nonlinear terms, we can get its equivalent small-signal ac model.

$$\hat{i}_a = I_c \hat{d} + D \hat{i}_c$$

$$\hat{v}_{cp} = V_{ap} \hat{d} + D \hat{v}_{ap}$$

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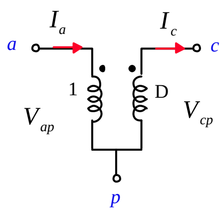
PWM Switch Model

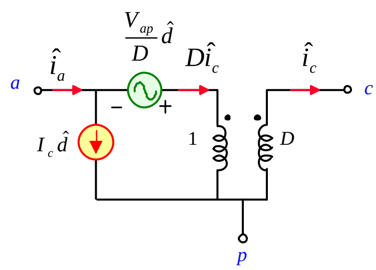
DC Model:

$$\begin{cases} I_a = DI_c \\ V_{cp} = DV_{ap} \end{cases}$$

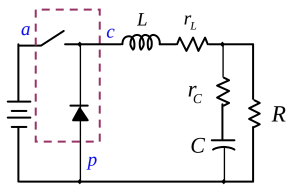


AC Model:

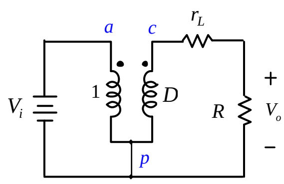
$$\begin{cases} \hat{i}_a = D\hat{i}_c + I_c\hat{d} \\ \hat{v}_{cp} = D\hat{v}_{ap} + V_{ap}\hat{d} \end{cases}$$



DC Analysis of a Buck Converter



DC Analysis



$$\frac{V_o}{V_i} = D \cdot \left(\frac{R}{R + r_L}\right)$$

AC Analysis of a Buck Converter

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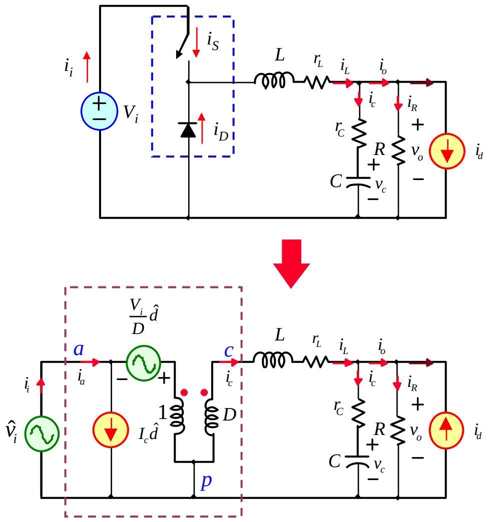
Small-signal equivalent circuit model of the buck converter.

Note:
In the following, the hat above the small signal variables are neglected for simplicity.



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AC Analysis: Control-to-Output Transfer Function

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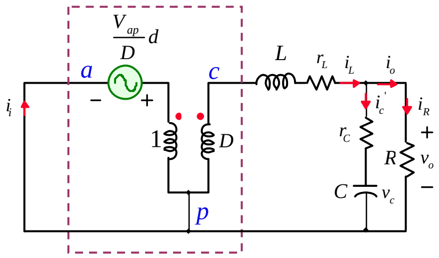

Short the unrelated voltage source and open the unrelated current source

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AC Analysis: Control-to-Output Transfer Function

$$v_{cp} = \frac{V_i}{D} d \cdot D$$

$$v_o = v_{cp} \frac{R//(r_C + 1/sC)}{(r_L + sL) + R//(r_C + 1/sC)}$$

$$= V_i d \frac{R(1 + s r_C C)}{s^2 (R + r_C) LC + s(r_L r_C C + r_L RC + L + r_C RC) + (R + r_L)}$$

$$G_d(s) = \frac{v_o(s)}{d(s)} = V_i \frac{R(1 + s r_C C)}{s^2 RLC + s(r_L r_C C + L + r_L RC + r_C RC) + (R + r_L)}$$

if $r_L << R$ and $r_L r_C$ can be neglected, the above eq. can be simplified as:

$$G_d(s) = \frac{v_o(s)}{d(s)} = \frac{V_i}{LC} \cdot \frac{1 + s r_C C}{s^2 + s(\frac{1}{RC} + \frac{r_L + r_C}{L}) + \frac{1}{LC}}$$

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AC Analysis: Line-to-Output Transfer Function

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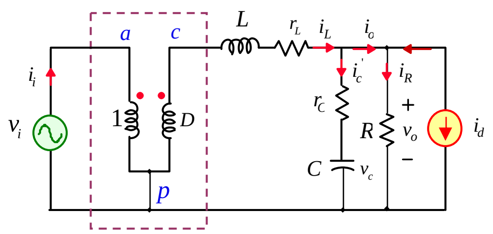

$$R//(r_C + 1/sC) = \frac{R(r_C + 1/sC)}{R + r_C + 1/sC}$$

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AC Analysis: Line-to-Output Transfer Function

$$v_{cp} = v_i D$$

$$v_o = v_{cp}\frac{R//(r_C + 1/sC)}{(r_L + sL) + R//(r_C + 1/sC)} = v_i D\frac{\dfrac{R(r_C + 1/sC)}{R + r_C + 1/sC}}{(r_L + sL) + \dfrac{R(r_C + 1/sC)}{R + r_C + 1/sC}}$$

$$= v_i D\frac{R(r_C + 1/sC)}{(r_L + sL)(R + r_C + 1/sC) + R(r_C + 1/sC)} = v_i D\frac{R(1 + sr_C C)}{(r_L + sL)(sRC + sr_C C + 1) + R(sr_C C + 1)}$$

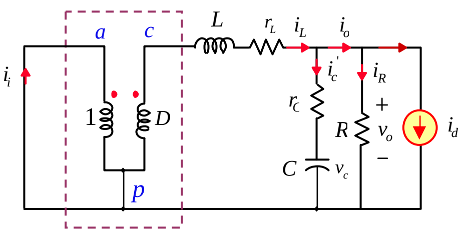
$$= v_i D\frac{R(1 + sr_C C)}{s^2(R + r_C)LC + s(r_L r_C C + r_L RC + L + r_C RC) + (R + r_L)}$$

$$G_v(s) = \frac{v_o(s)}{v_i(s)} = D\frac{R(1 + sr_C C)}{s^2 RLC + s(r_L r_C C + L + r_L RC + r_C RC) + (R + r_L)}$$

if $r_L << R$ and $r_L r_C$ can be neglected, the above equation can be simplified as:

$$G_v(s) = \frac{v_o(s)}{v_i(s)} = \frac{D}{LC}\cdot\frac{1 + sr_C C}{s^2 + s(\dfrac{1}{RC} + \dfrac{r_L + r_C}{L}) + \dfrac{1}{LC}}$$

AC Analysis: Disturbance-to-Output Transfer Function



AC Analysis: Disturbance-to-Output Transfer Function

$$\frac{v_o}{i_d} = R//(r_C+1/sC)//(r_L+sL) = \frac{R(r_C+1/sC)}{R+r_C+1/sC}//(r_L+sL)$$

$$= \frac{\frac{R(r_C+1/sC)}{R+r_C+1/sC}(r_L+sL)}{\frac{R(r_C+1/sC)}{R+r_C+1/sC}+r_L+sL} = \frac{R(r_C+1/sC)(r_L+sL)}{R(r_C+1/sC)+(R+r_C+1/sC)(r_L+sL)}$$

$$= \frac{R(sr_CC+1)(r_L+sL)}{R(sr_CC+1)+(sRC+sr_CC+1)(r_L+sL)} = \frac{R\left[s^2r_CLC+sr_Lr_CC+sL+r_L\right]}{sr_CRC+R+(sRC+sr_CC+1)(r_L+sL)}$$

$$= \frac{R\left[s^2r_CLC+sr_Lr_CC+sL+r_L\right]}{sr_CRC+R+s^2RLC+s^2r_CLC+sr_Cr_LC+sr_LRC+sL+r_L}$$

$$\frac{v_o}{i_d} = \frac{R\left[s^2r_CLC+s(L+r_Lr_CC)+r_L\right]}{s^2(R+r_C)LC+s\left[L+(r_C+r_L)RC+r_Cr_LC\right]+(R+r_L)}$$

AC Analysis: Disturbance-to-Output Transfer Function

if $r_L \ll R$ and r_Lr_C can be neglected, the above equation can be simplified as:

$$Z_o(s) = \frac{v_o(s)}{i_d(s)} = \frac{R\left[s^2r_CLC+sL+r_L\right]}{s^2RLC+s\left[L+(r_C+r_L)RC\right]+R}$$

$$= \frac{s^2r_CLC+sL+r_L}{s^2LC+s\left[\frac{L}{R}+(r_C+r_L)C\right]+1} = \frac{s^2r_C+s\frac{1}{C}+\frac{r_L}{LC}}{s^2+s\left[\frac{1}{RC}+\frac{r_C+r_L}{L}\right]+\frac{1}{LC}}$$

The Output Impedance of a Buck Converter is:

$$Z_o(s) = \frac{v_o(s)}{i_d(s)} = \frac{s^2r_C+s\frac{1}{C}+\frac{r_L}{LC}}{s^2+s\left[\frac{1}{RC}+\frac{r_C+r_L}{L}\right]+\frac{1}{LC}}$$

AC Analysis: Disturbance-to-Output Transfer Function

$$Z_o(s)=\frac{v_o(s)}{i_d(s)}=\frac{s^2 r_C+s\frac{1}{C}+\frac{r_L}{LC}}{s^2+s\left[\frac{1}{RC}+\frac{r_C+r_L}{L}\right]+\frac{1}{LC}}$$

$$\omega_o^2=\frac{1}{LC} \quad \Rightarrow \quad \omega=\frac{1}{\sqrt{LC}}$$

$$\frac{\omega_o}{Q}=\frac{1}{RC}+\frac{r_C+r_L}{L}$$

$$Q=\frac{1}{\omega_o}\frac{\omega_o^2}{\frac{1}{RC}+\frac{r_C+r_L}{L}}=\frac{1}{\omega_o}\frac{1}{LC\left(\frac{1}{RC}+\frac{r_C+r_L}{L}\right)}=\frac{1}{\omega_o}\frac{1}{\frac{L}{R}+(r_C+r_L)L}$$

AC Analysis: Disturbance-to-Output Transfer Function

$$Q=\frac{1}{\omega_o}\frac{1}{\frac{L}{R}+(r_C+r_L)L}$$

$$Z_{op}(s)=s^2+s\left[\frac{1}{RC}+\frac{r_C+r_L}{L}\right]+\frac{1}{LC}$$

$$=s^2+s\frac{\omega_o}{Q}+\omega_o^2$$

$$Z_{oq}(s)=s^2 r_C+s\frac{1}{C}+\frac{r_L}{LC}$$

$$=\frac{r_L}{LC}\left(\frac{r_C}{r_L}LCs^2+\frac{L}{r_L}s+1\right)$$

$$=R_{eq}\omega_o^2\left(\frac{1}{\omega_1^2}s^2+\frac{1}{\omega_1 Q_1}s+1\right)$$

$$\Delta s=s^2+s\frac{\omega_o}{Q}+\omega_o^2$$

$$R_{eq}=r_L$$

$$\omega_1=\sqrt{\frac{r_L}{r_C}}\frac{1}{\sqrt{LC}}$$

$$Q_1=\frac{1}{\omega_1}\frac{r_L}{L}$$

AC Analysis: Disturbance-to-Output Transfer Function

The exact output impedance is derived as:

$$\frac{v_o}{i_d} = \frac{R\left[s^2 r_C LC + s(L + r_L r_C C) + r_L\right]}{s^2(R + r_C)LC + s\left[L + (r_C + r_L)RC + r_C r_L C\right] + (R + r_L)}$$

$$= \frac{s^2 r_C LC + s(L + r_L r_C C) + r_L}{s^2(1 + \frac{r_C}{R})LC + s\left[\frac{L}{R} + (r_C + r_L)C + r_C r_L \frac{C}{R}\right] + (1 + \frac{r_L}{R})}$$

$$= \frac{s^2 r_C + s(\frac{1}{C} + \frac{r_L r_C}{L}) + \frac{r_L}{LC}}{s^2(1 + \frac{r_C}{R}) + s\left[\frac{1}{RC} + \frac{1}{L}(r_C + r_L + \frac{r_L r_C}{R})\right] + (1 + \frac{r_L}{R})\frac{1}{LC}}$$

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AC Analysis: Disturbance-to-Output Transfer Function

$$Z_{oq}(s) = s^2 r_C + s\left(\frac{1}{C} + \frac{r_L r_C}{L}\right) + \frac{r_L}{LC} = \frac{r_L}{LC}\left(\frac{r_C}{r_L} LCs^2 + (\frac{L}{r_L} + r_C C)s + 1\right)$$

$$= R_{eq}\omega_o^2\left(\frac{1}{\omega_1^2}s^2 + \frac{1}{\omega_1 Q_1}s + 1\right)$$

$$R_{eq} = r_L$$

$$\omega_1 = \sqrt{\frac{r_L}{r_C}}\frac{1}{\sqrt{LC}}$$

$$Q_1 = \frac{1}{\omega_1}\frac{1}{\frac{L}{r_L} + r_C C}$$

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Output Impedance of the Buck Converter

Output Impedance of the Buck Converter:

$$Z_o(s)=\frac{v_o(s)}{i_d(s)}=\frac{s^2 r_C + s(\frac{1}{C}+\frac{r_L r_C}{L})+\frac{r_L}{LC}}{s^2(1+\frac{r_C}{R})+s\left[\frac{1}{RC}+\frac{1}{L}(r_C+r_L+\frac{r_L r_C}{R})\right]+(1+\frac{r_L}{R})\frac{1}{LC}}$$

$$Z_o(s)=\frac{v_o(s)}{i_d(s)}=R_{eq}\frac{1+\frac{s}{\omega_1 Q_1}+(\frac{s}{\omega_1})^2}{\Delta s/\omega_o^2}$$

$$\Delta s = s^2+\frac{\omega_o}{Q}s+\omega_o^2 , \qquad R_{eq}=r_L$$

$$\omega_o=\frac{1}{\sqrt{LC}} , \qquad Q=\frac{1}{\omega_o}\frac{1}{\frac{L}{R}+(r_L+r_C)C}$$

$$\omega_1=\frac{1}{\sqrt{LC}}\sqrt{\frac{r_L}{r_C}} ; \qquad Q_1=\frac{1}{\omega_1}\frac{1}{\frac{L}{r_C}+r_L C}$$

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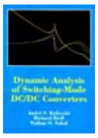
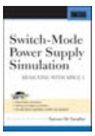
Recommended Books: Modeling and Simulation



Dynamic Analysis of Switching-Mode DC/DC Converters,
Andre'S. Kislovski, Richard Redl, and Nathan O. Sokal,
Van Nostrand Reinhold, New York, 1991.



Switch-Mode Power Supply Simulation: Designing with SPICE 3
Steven M. Sandler,
McGraw-Hill Professional; 1 edition, Nov. 11, 2005.

Switch-Mode Power Supplies - SPICE Simulations and Practical Designs,
Christophe Basso,
McGraw-Hill, Feb. 1, 2008.

Complex Behavior of Switching Power Converters,
Chi Kong Tse, CRC Press, 2004.

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References: Modeling of DC-DC Converters

Recommended Readings

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[2] A. J. Forsyth and S. V. Mollov, "Modelling and control of DC-DC converters," *IEEE Power Engineering Journal*, vol. 12, no. 5, pp. 229-236, 1998.

[3] R. D. Middlebrook, "Small-signal modeling of PWM switched-mode power converters," *IEEE Proc.* vol. 76, no. 4, pp. 343-354, April 1988.

[4] R. D. Middlebrook and S. C'uk, "A general unified approach to modeling switching converter power stages," *IEEE PESC Conf. Rec.*, pp. 18-34, 1976. [Pioneer paper]

SPS Modeling and Control Loop Design

[5] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part I: Continuous Conduction Mode," *IEEE Trans. on Aero. and Electron. Sys.*, vol. 26, no. 3, pp. 490-496, May 1990.

[6] V. Vorperian, "Simplified Analysis of PWM Converters Using Model of PWM Switch Part II: Discontinuous Conduction Mode," *IEEE Trans. on Aero. and Electron. Sys.*, vol. 26, no. 3, pp. 497-505, May 1990.

[7] V. Voperian, R. Tymerski, and F. C. Lee, "Equivalent circuit models for resonant and PWM switches," IEEE Trans. on Power Electronics, vol. 4, no. 2, pp. 205-214, April 1989.

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Modeling of Switching Converters in DCM Operation

[11] D. Maksimovic and S. Cuk, "A unified analysis of PWM converters in discontinuous modes," IEEE Trans. Power Electron., vol. 6, pp. 476–490, May 1991.

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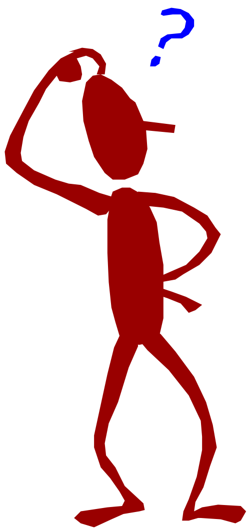

Any Questions ???

Questions inspire effective learning!

學習的關鍵

- 記筆記
- 問問題

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