2nd-Order System

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Lab808: 電力電子系統與晶片實驗室
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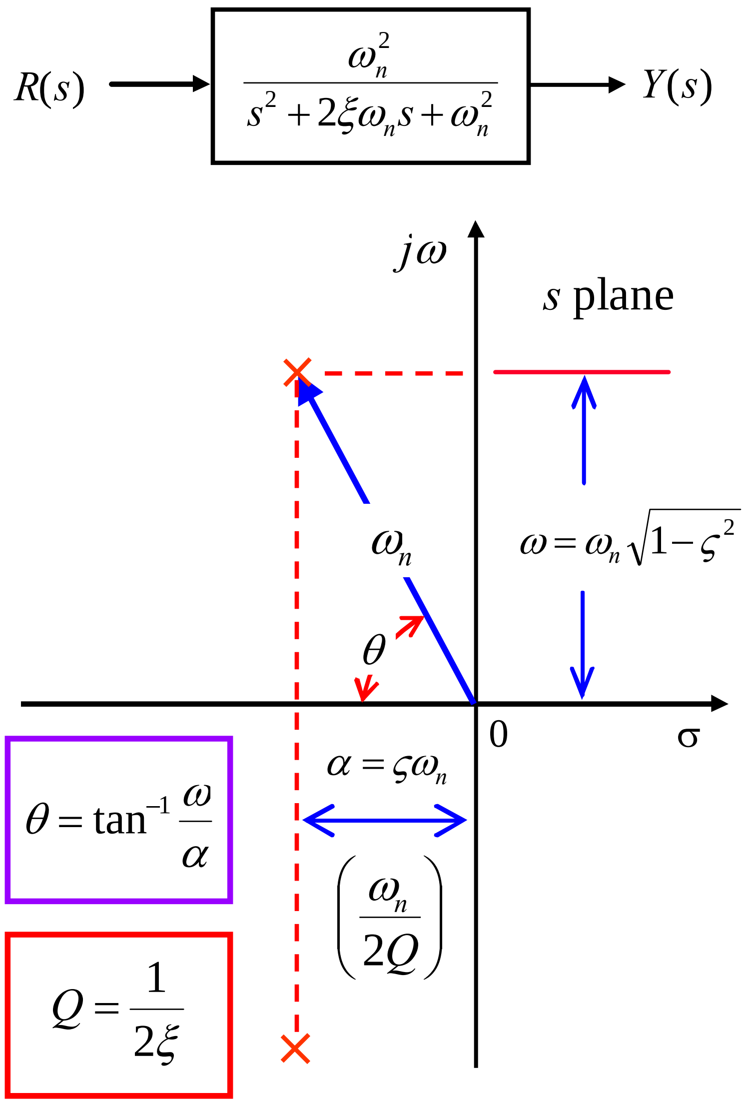
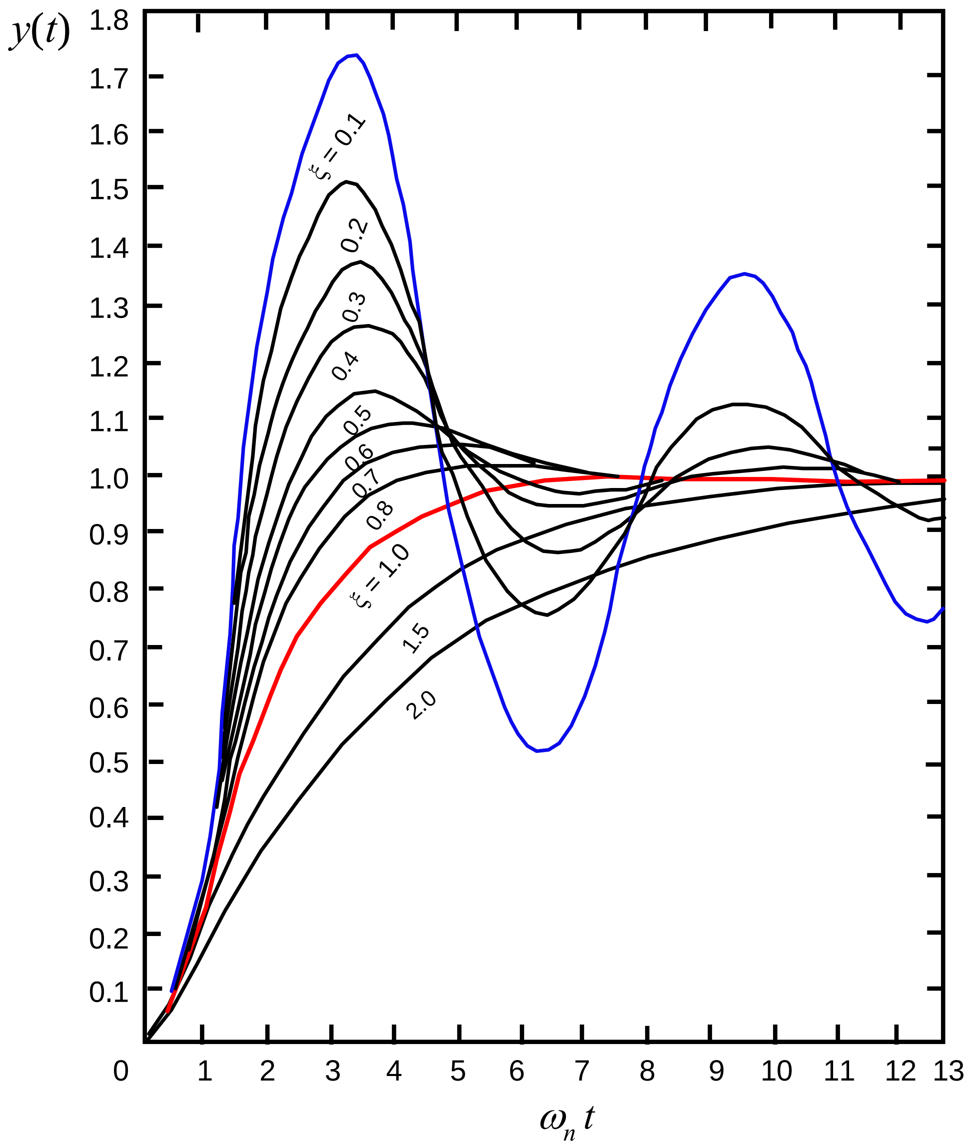
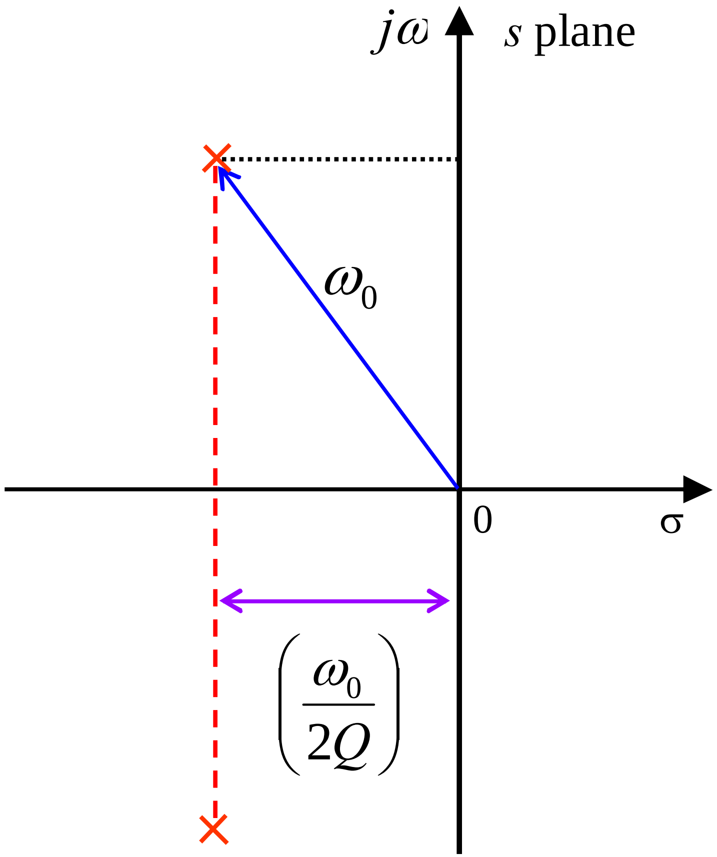
Normalized Time Responses

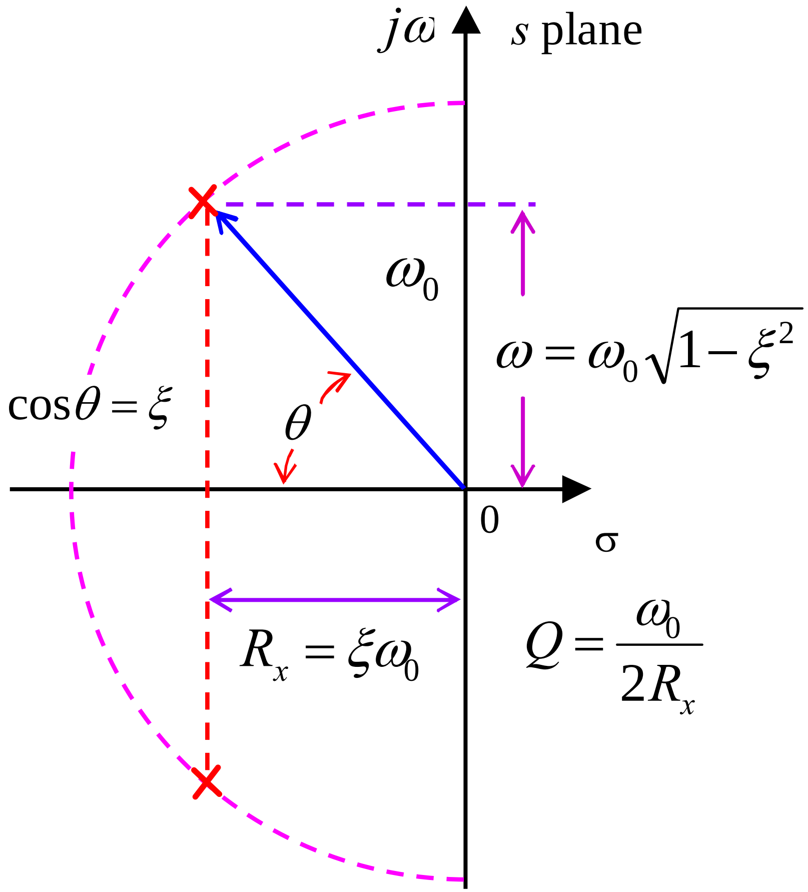
Step Responses of a Second-Order System





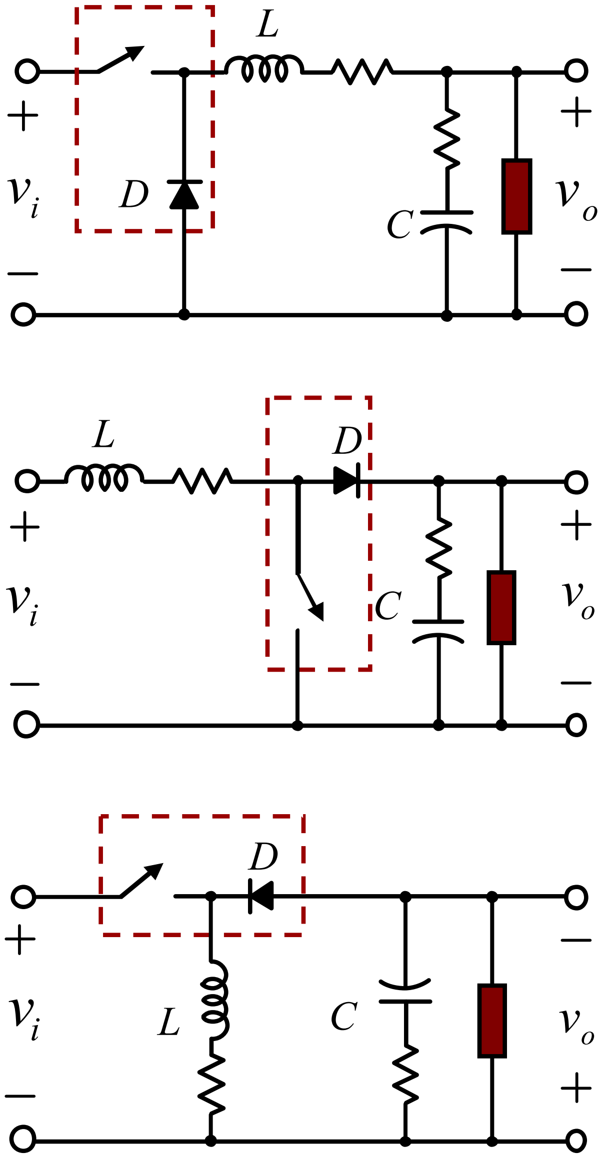
Damping Ratio and Pole Quality Factor



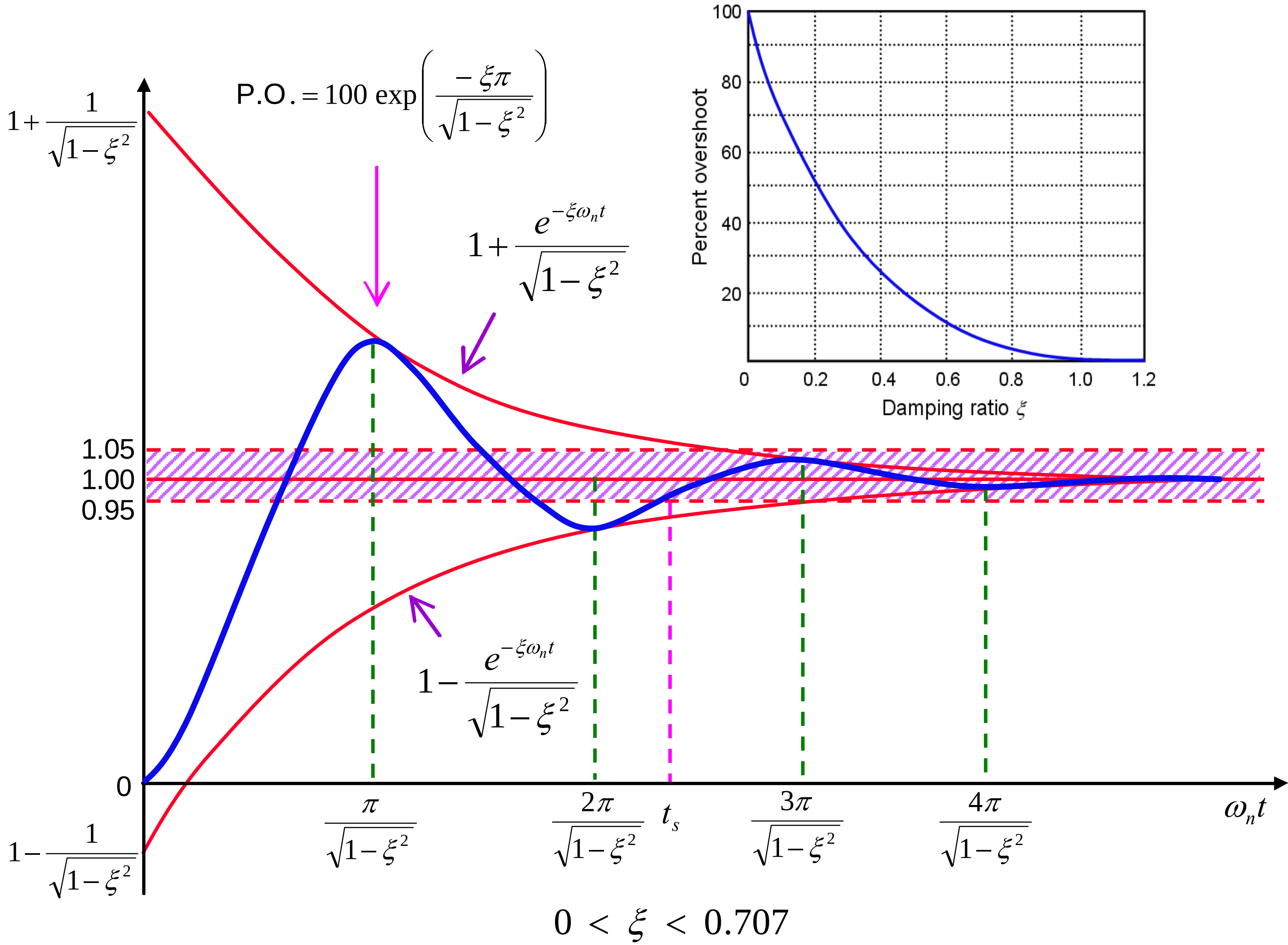


Pole Quality Factor $Q = \frac{1}{2\xi} = \frac{1}{2 \cdot \text{Damping Factor}}$

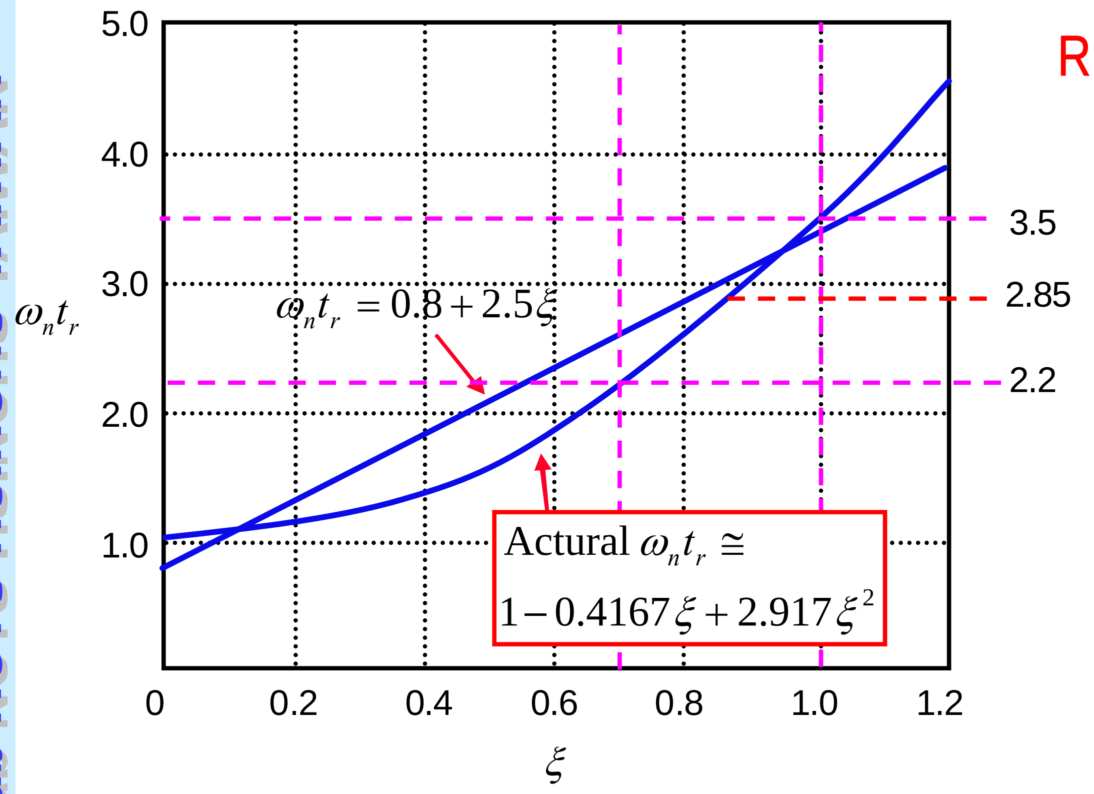
Settling Time of a Unit Step Response for a Second-Order Under Damped System



What is the step response for a duty ratio change?



Normalized Rise Time vs. Damping Ratio



Rise Time, Bandwidth, and Damping Ratio

$$\omega_n t_r \cong 1 - 0.4167\,\xi + 2.917\,\xi^2$$

$\xi = 0.707 \Rightarrow \omega_n t_r = 2.163 \approx 2.2$

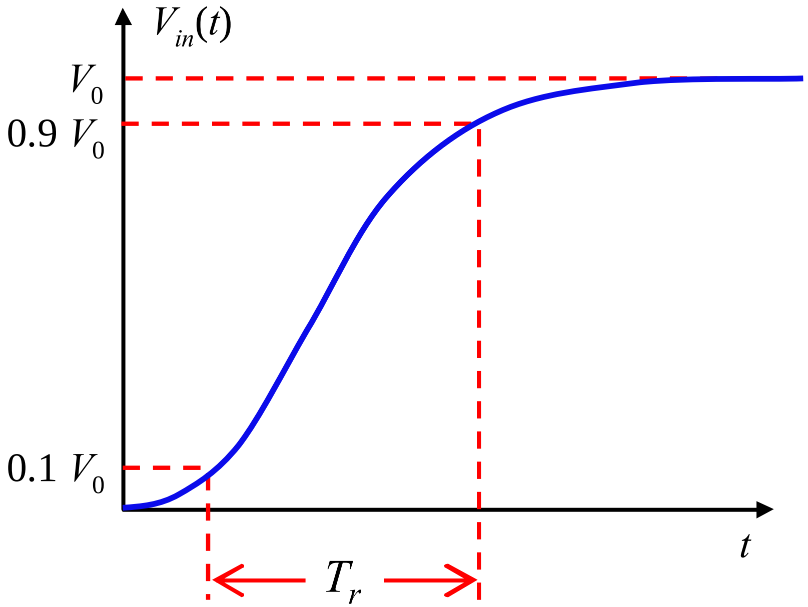
$\xi = 1.0 \Rightarrow \omega_n t_r = 3.5$

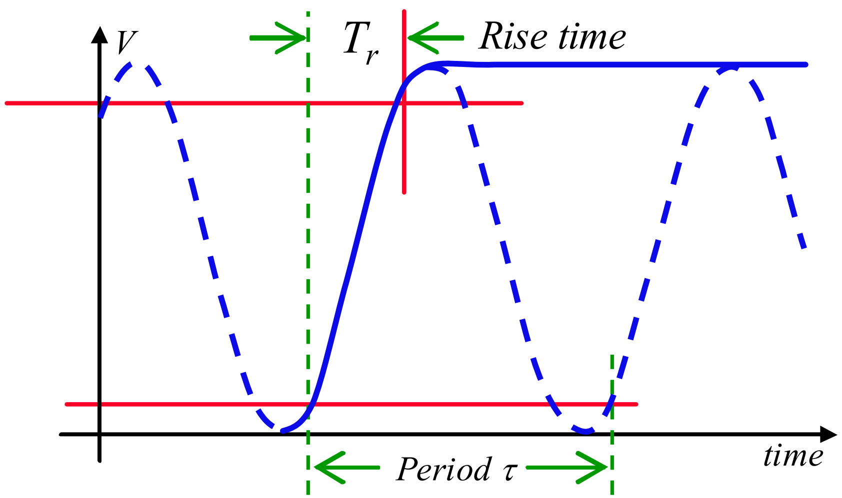
$$\omega_n = \frac{3.5}{t_r} = 2\pi f_n$$

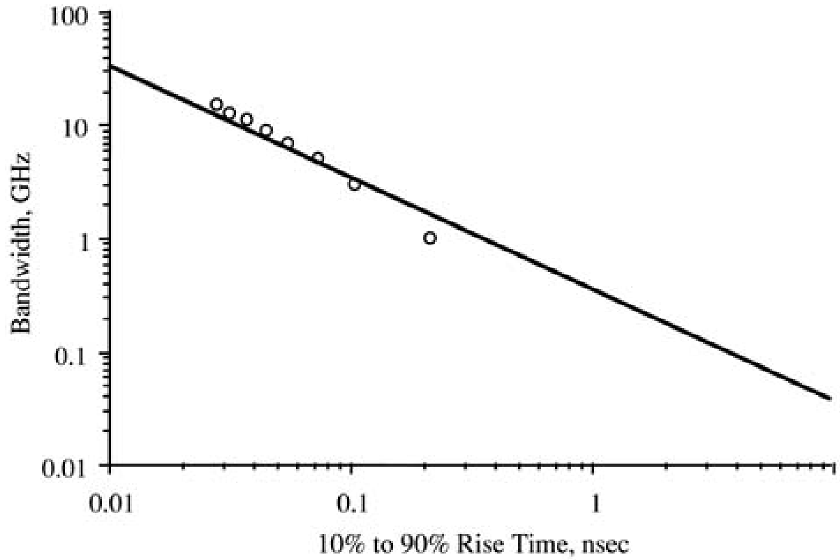
$$f_n = \frac{3.5}{2\pi t_r} = \frac{0.557}{t_r}$$

- t_r and t_d are proportional to ξ and inversely proportional to ω_n.
- Increasing (decreasing) the natural undamped frequency ω_n will reduce (increase) t_r and t_d.

Rise Time and Bandwidth





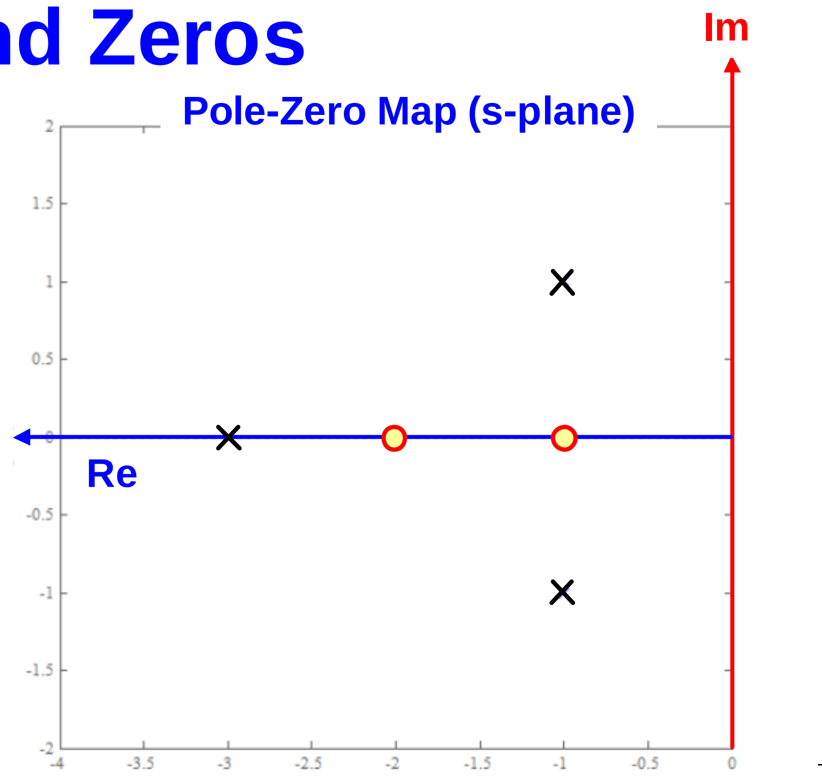


Empirical relationship between the bandwidth of a signal and its 10–90 rise time, as measured from a re-created ideal square wave with each harmonic added one at a time. Circles are the values extracted from the data; line is the approximation of BW = 0.35/rise time.

$$\text{BW} = \frac{0.35}{T_r}\ (rad/\text{sec})$$

The rise time of a pulse is approximately 35% of the period of the underlying sine wave.

Understanding Poles and Zeros



$$H(s) = \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} \Rightarrow H(s) = k\frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)(s-p_3)}$$

$$H(s) = k\frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{i=1}^{p}(s-p_i)}$$

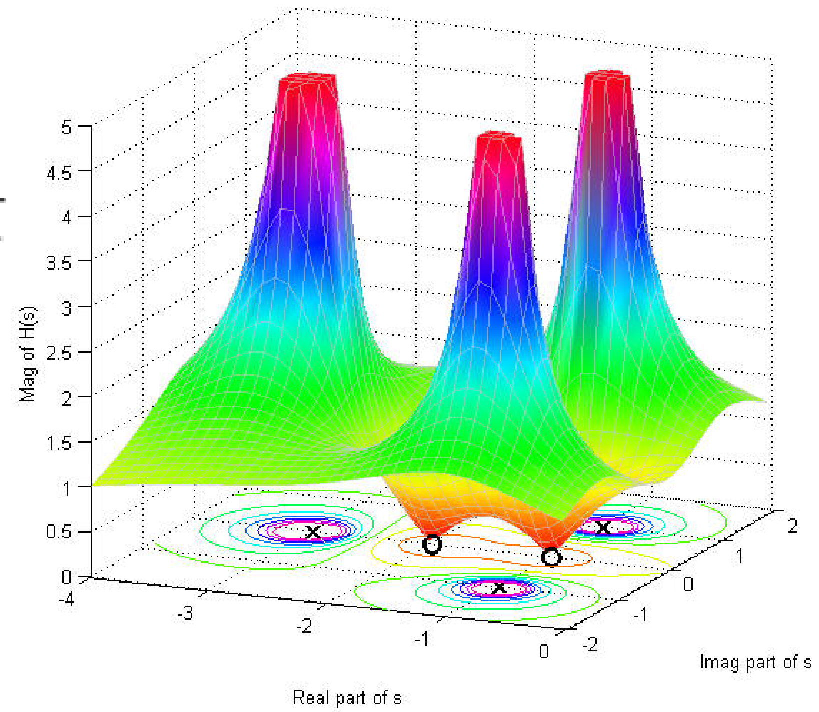
Zeros

Poles

$$H(s) = \frac{6s^2 + 18s + 12}{2s^3 + 10s^2 + 16s + 12}$$

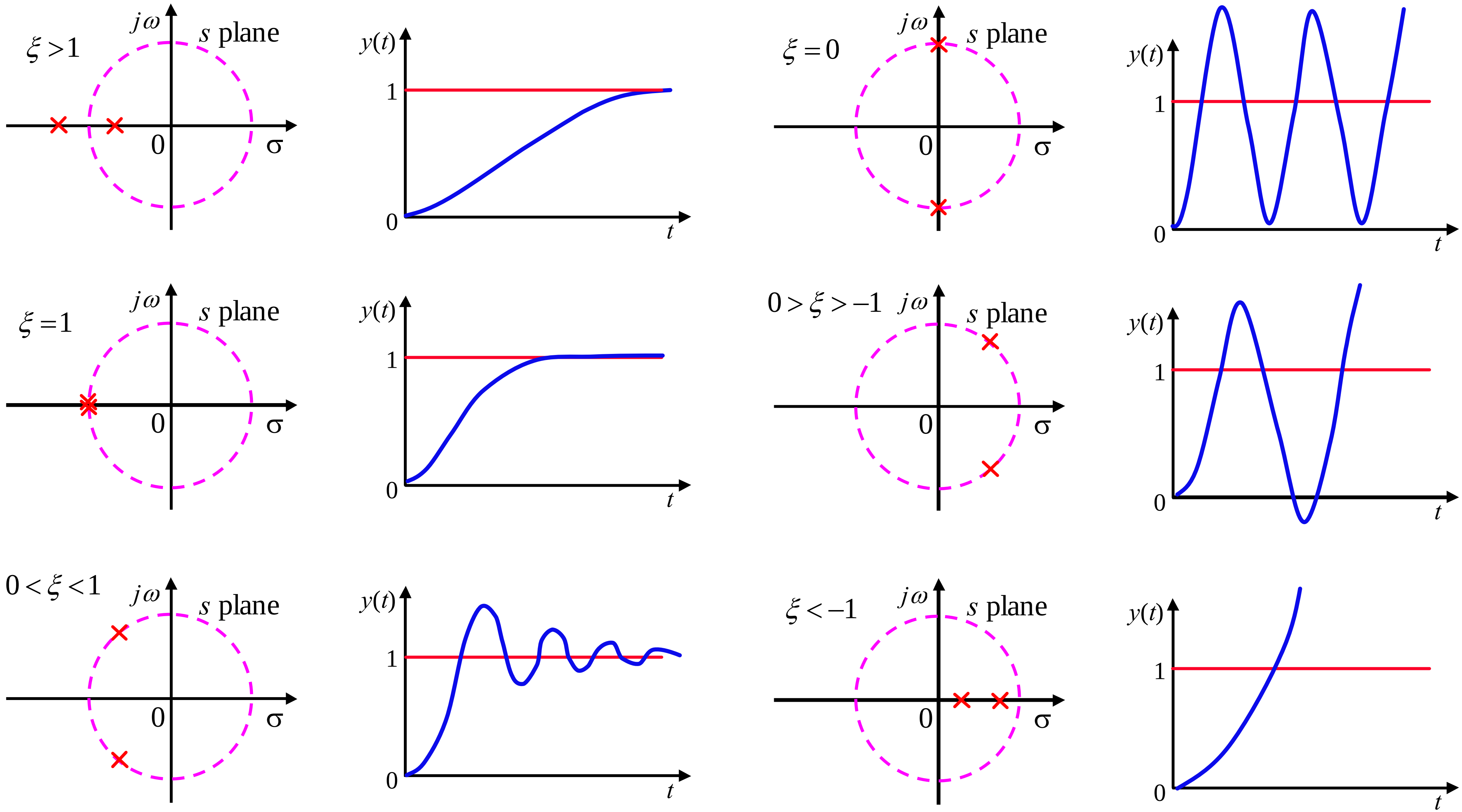
$$H(s) = \frac{6}{2}\frac{s^2 + 3s + 2}{s^3 + 5s^2 + 8s + 6} = 3\frac{(s+1)(s+2)}{(s+1-j)(s+1-j)(s+3)}$$

$$j = \sqrt{-1}$$

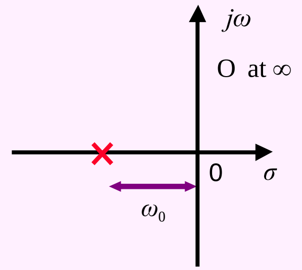
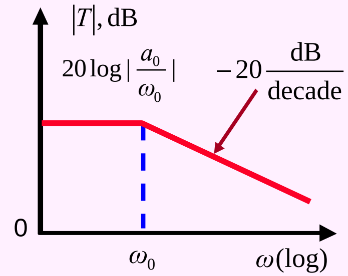
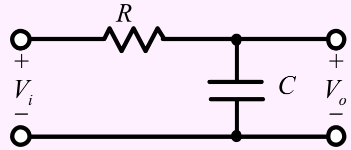
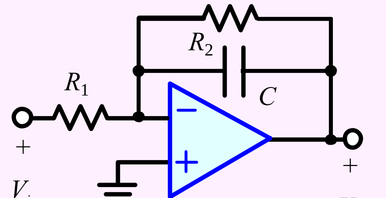
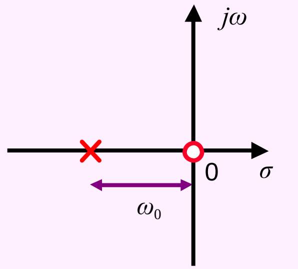
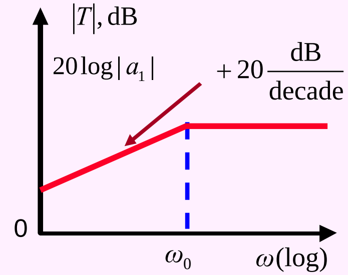
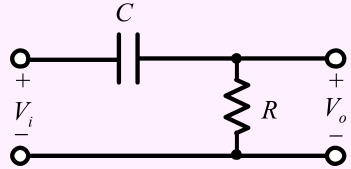
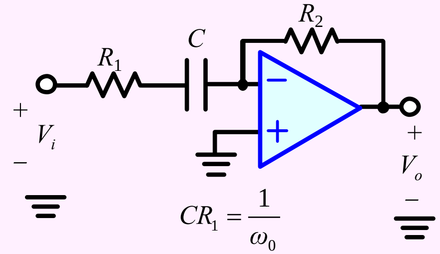
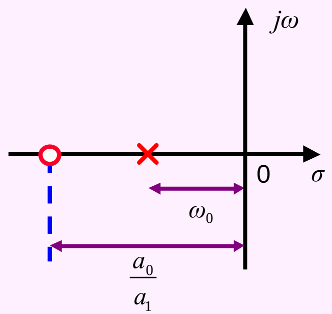
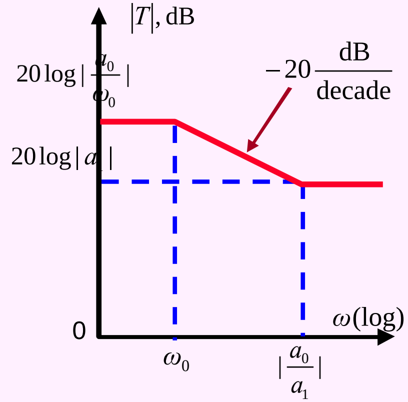
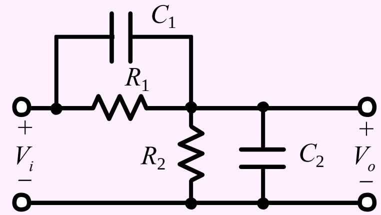
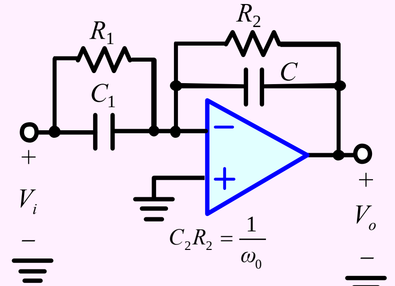


Understanding Poles and Zeros,
Analysis and Design of Feedback Control Systems,
MIT course note.

Step Time Responses for 2nd-Order System

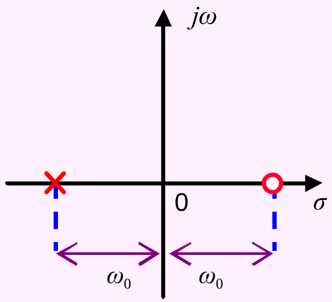
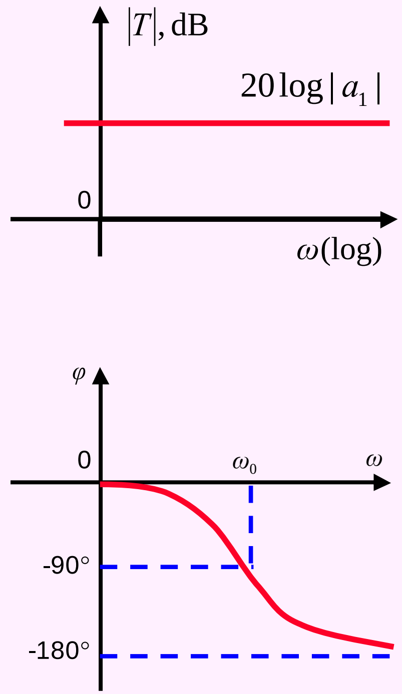
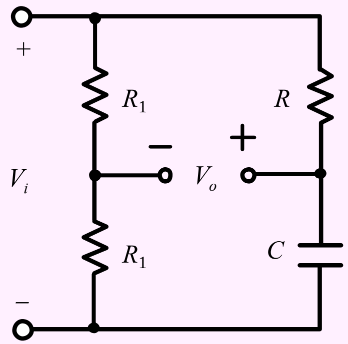
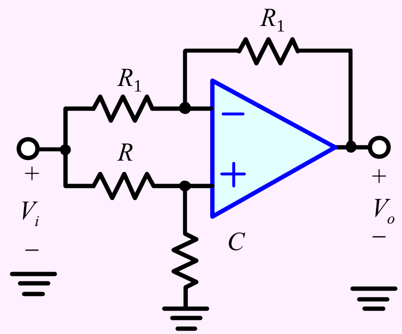


First-Order Filters

Filter Type and T(s)	s-Plane Singularities	Bode Plot for $\lvert T \rvert$	Passive Realization	Op Amp-RC Realization
(a) Low-Pass (LP) $T(s)=\frac{a_0}{s+\omega_0}$			 $CR=\frac{1}{\omega_0}$ dc gain $=1$	 $CR_2=\frac{1}{\omega_0}$ dc gain $=-\frac{R_2}{R_1}$
(b) High-Pass (HP) $T(s)=\frac{a_1 s}{s+\omega_0}$			 $CR=\frac{1}{\omega_0}$ High-frequency $=1$	 $CR_1=\frac{1}{\omega_0}$ High-frequency gain $=-\frac{R_2}{R_1}$
(c) Central $T(s)=\frac{a_1 s+a_0}{s+\omega_0}$			 $(C_1+C_2)(R_1+R_2)=\frac{1}{\omega_0}$ $C_1R_1=\frac{a_0}{a_1}$ dc gain $=\frac{R_2}{R_1+R_2}$ HF gain $=\frac{C_1}{C_1+C_2}$	 $C_2R_2=\frac{1}{\omega_0}$ $C_1R_1=\frac{a_1}{a_0}$ dc gain $=-\frac{R_2}{R_1}$ HF gain $=-\frac{C_1}{C_2}$

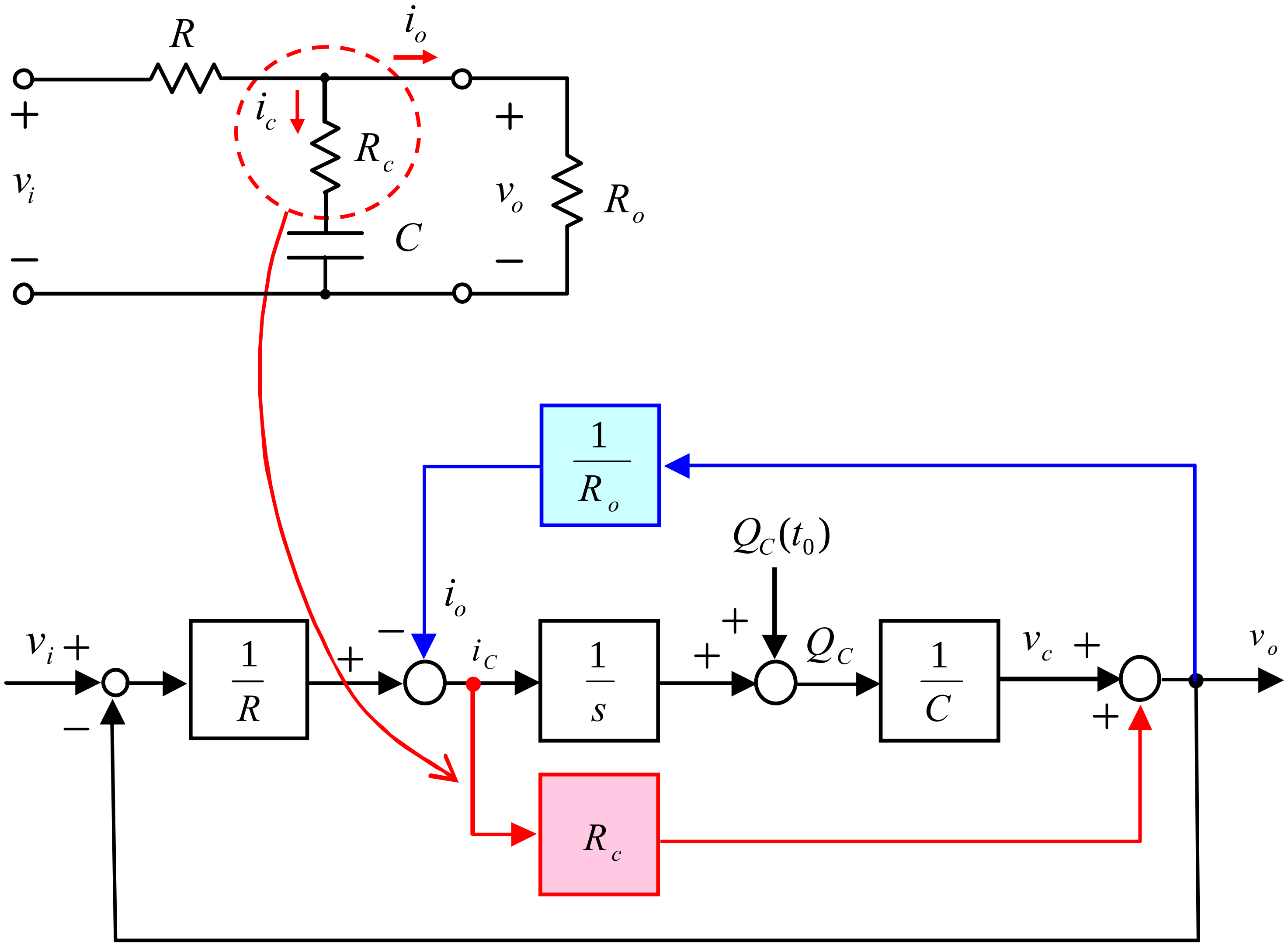
All-Pass Filter

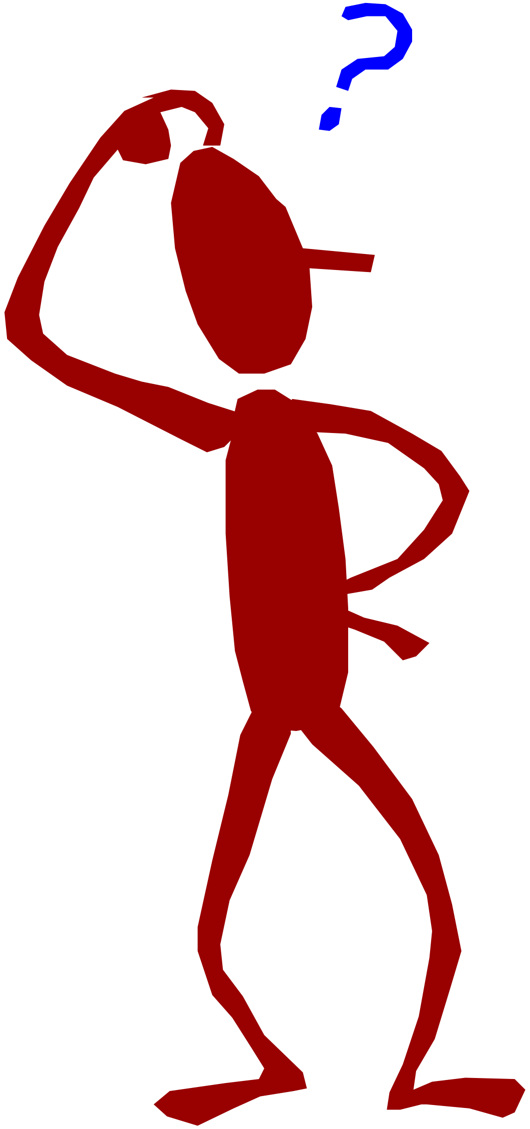
All-pass filter characterizes a constant transmission at all frequencies and the transmission zero and the natural mode are symmetrically located relative to the $j\omega$-axis.

T(s)	Singularities	$\lvert T\rvert$ and ϕ	Passive Realization	Op Amp-RC Realization
$T(s) = -a_1 \frac{s-\omega_0}{s+\omega_0}$ $a_1 > 0$			 $CR = \frac{1}{\omega_0}$ Flat gain$(a_1) = 0.5$	 $CR = \frac{1}{\omega_0}$ Flat gain$(a_1) = 1$

Introduces phase lag at specified frequency.

A Zero Means a Feed-Forward Path





Any Questions ???

Questions inspire effective learning!

學習的關鍵

- 記筆記
- 問問題

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