808-PowerLab NCTU HSINCHU TAIWAN

Right-Half-Plane (RHP) Zero

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2009年6月4日

Lab808: 電力電子系統與晶片實驗室
Power Electronic Systems & Chips, NCTU, TAIWAN
台灣新竹•交通大學•電機與控制工程研究所

Zero in the Boost and Buck/Boost Converters (CCM)

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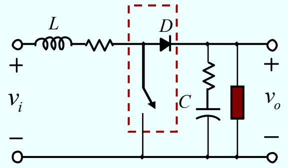
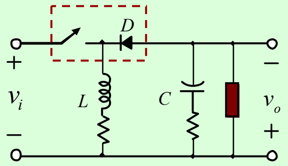


- **Boost**

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_i}{D'^2}\frac{(1+r_C Cs)(1-sL/(D'^2R))}{(\frac{s}{\omega_o})^2+\frac{1}{\omega_o Q}s+1}$$

$$\omega_o = \frac{D}{\sqrt{LC}}$$

$$Q = \frac{D}{\omega_o}\frac{1}{\frac{L}{D\ R}+\frac{r_L C}{D}+r_C C}$$



- **Buck/Boost**

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_i}{D'^2}\frac{(1+r_C Cs)(1-sDL/(D'^2R))}{(\frac{s}{\omega_o})^2+\frac{1}{\omega_o Q}s+1}$$

$$\omega_o = \frac{D}{\sqrt{LC}}$$

$$Q = \frac{D}{\omega_o}\frac{1}{\frac{L}{(D)^2R}+(\frac{r_C}{D}+\frac{r_L}{(D)^2})C}$$

- Why the RHP zero will occur?
- Where is the RHP zero?
- What is the effect of the RHP zero?
- Can the RHP zero be eliminated?

Right-Half-Plane (RHP) Zero

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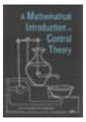
Right-Half-Plane Zero (of DC-DC Converters) - A simplified explanation, Lloyd Dixon, 1984, (slup084)

Understanding the Right-Half-Plane Zero,
May 1, 2009 12:00 PM
CHRISTOPHE BASSO, Director, Product Application Engineering, ON Semiconductor, Phoenix

Tricks of the Trade - understanding the right-half-plane zero in small-signal DC-DC converter models,
IEEE Power Electronics Society NEWSLETTER, January 2001.



A Mathematical Introduction to Control Theory,
Shlomo Engelber, Imperial College Press,
pp. 147, The Effect of Zeros in the Right Half-Plane

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RHP Zero of Boost and Buck-Boost Converters

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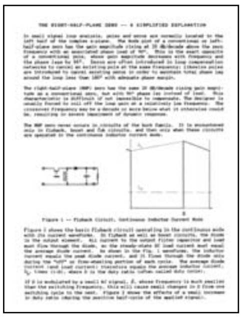
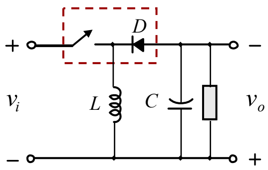
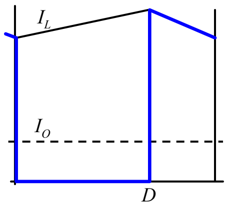
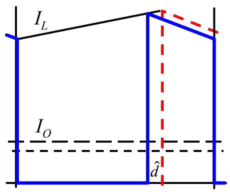


Right-Half-Plane Zero - A simplified explanation, Lloyd Dixon, 1984, (slup084)





Flyback Circuit, Continuous Inductor Current Mode



Switchmode power supply handbook
Keith H. Billings

Incremental changes in duty ratio.

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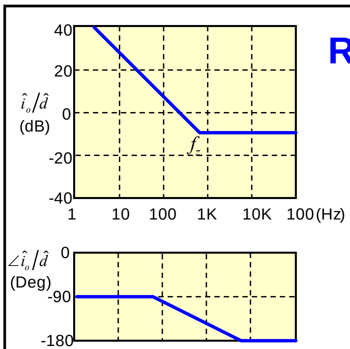
Right-Half-Plane (RHP) Zero



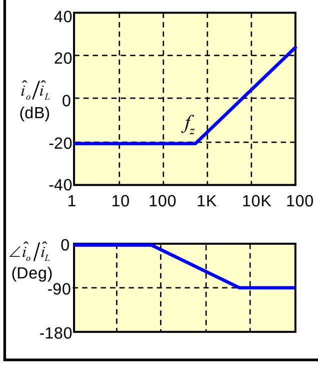
$$V_L = V_i D - V_O(1-D) = (V_i + V_O)D - V_O$$

$$\hat{V}_L = (V_i + V_O)\hat{d} + \hat{V}_O(1-D) = (V_i + V_O)\hat{d}$$

$$i_O = -j\frac{(V_i+V_O)(1-D)}{\omega L}\hat{d} - I_L\hat{d} = -j\frac{V_i}{\omega L}\hat{d} - I_L\hat{d}$$

$$\omega_z = \frac{R_O V_i(1-D)}{L V_O} = \frac{R_O(1-D)^2}{LD} = \frac{R_O V_i^2}{L V_O(V_i+V_O)}$$

Frequency response of RHP zero in voltage-mode control



$$\hat{i}_O = \hat{i}_L(1-D) - j\frac{\omega L I_L}{(V_i+V_O)}\hat{i}_L = \frac{V_i}{(V_i+V_O)}\hat{i}_L - j\frac{\omega L I_L}{(V_i+V_O)}\hat{i}_L$$

The RHP zero is still present with current mode control!

Frequency response of RHP zero in current-mode control

RHP Zero of CCM Buck-Boost (Flyback) Converter

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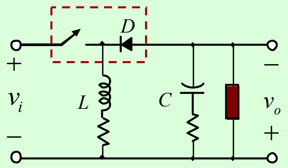
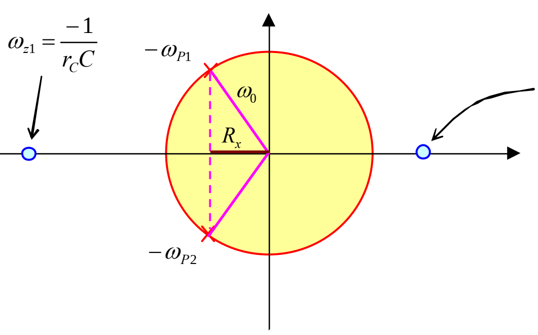


- **Buck/Boost**

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_i}{D^2}\frac{(1+r_C Cs)(1-sDL/(D'^2R))}{(\frac{s}{\omega_o})^2 + \frac{s}{\omega_o Q} + 1}$$

$$\omega_o = \frac{D}{\sqrt{LC}}$$

$$Q = \frac{D}{\omega_o}\frac{1}{\frac{L}{(D)^2R} + (\frac{r_C}{D} + \frac{r_L}{(D)^2})C}$$



$$i_O = \hat{d}\left(\frac{V_i}{s\omega L} - I_L\right)$$

$$\omega_{z2} = \frac{V_i}{L I_L} = \frac{R_O(1-D)^2}{LD}$$

$$Q = \frac{\omega_0}{2R_x}$$

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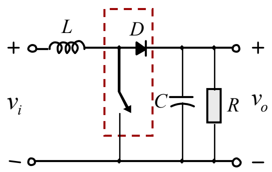
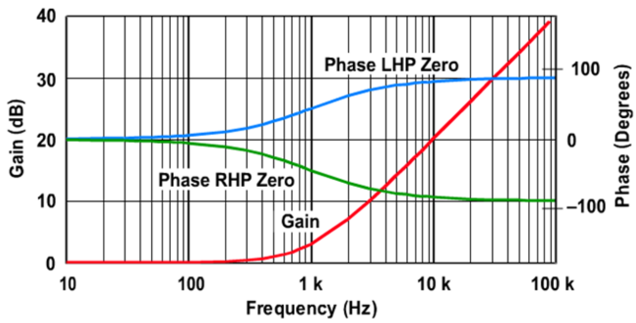
RHP Zero of CCM Boost Converter

- The effect of any control action during the ON time is delayed until the switch is turned FF.
- Output response is initially in the opposite direction of the desired correction.

RHP Zero





Brian Lynch, **Under the Hood of a DC-DC Boost Converter**, 2008-09 Power Supply Design Seminar - SEM1800.

Compensation of RHP Zero

A current mode control boost regulator has an inherent right half plane zero (RHP zero). This zero has the effect of a zero in the gain plot, causing an imposed +20dB/decade on the rolloff, but has the effect of a pole in the phase, subtracting another 90° in the phase plot. This can cause undesirable effects if the control loop is influenced by this zero.

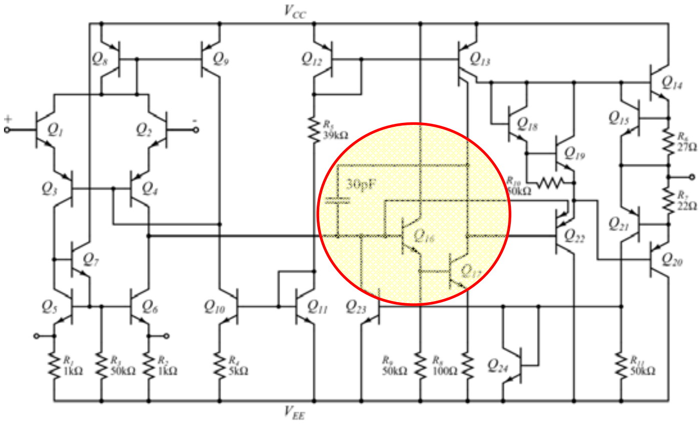
To ensure the RHP zero does not cause instability issues, the control loop should be designed to have a bandwidth of ½ the frequency of the RHP zero or less. This zero occurs at a frequency of:

$$\text{RFPzero} = \frac{V_{OUT}(D')^2}{2\pi I_{LOAD} L} \text{(in Hz)}$$

where I_{LOAD} is the maximum load current.

REF: LM5000 High Voltage Switch Mode Regulator

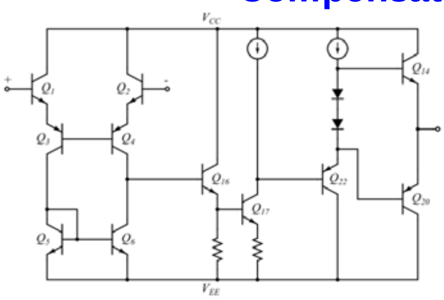
Compensation of OP-Amp uA741



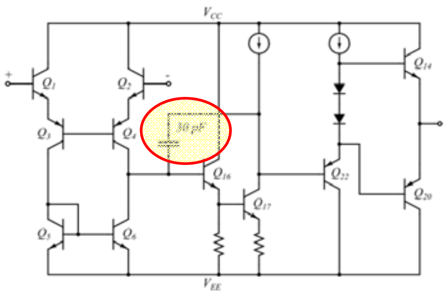
Complete Fairchild µA741 schematic. The primary signal path is comprised of two gain stages. The first gain stage is a differential pair with active load (transistors Q_1–Q_6). The second gain stage is a common-emitter amplifier (transistors Q_{16} and Q_{17}). The output buffer is a push-pull emitter follower (transistors Q_{14}, Q_{20}, and Q_{22}).

Kent H. Lundberg, "Internal and External Op-Amp Compensation: A Control-Centric Tutorial," American Control Conference, 2004.

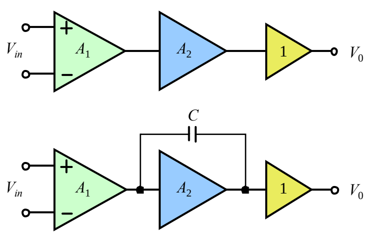
Compensation for OP-AMP



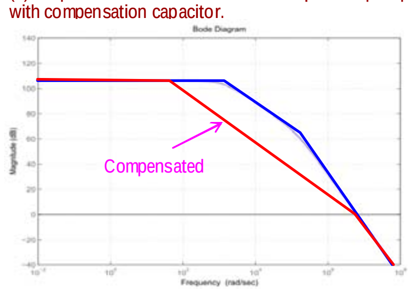
(a) Simplified schematic of the uncompensated Fairchild µA741 op amp.



(b) Simplified schematic of the Fairchild µA741 op amp with compensation capacitor.

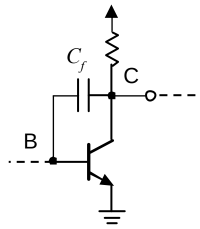


(d) Equivalent circuit of uncompensated and compensated OP-Amp.

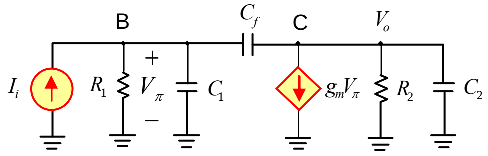


(d) Frequency response of uncompensated and compensated op amp. The two low-frequency poles in the uncompensated transfer function severely degrade the phase margin at crossover.

RHP Zero of a Common-Emitter Amp



(a) Common-emitter amp.



(b) Equivalent circuit

$$\frac{V_o}{I_i}=\frac{(sC_f-g_m)R_1R_2}{1+s[C_1R_1+C_2R_2+C_f(g_mR_1R_2+R_1+R_2)]+s^2[C_1C_2+C_f(C_1+C_2)]R_1R_2}$$

There is a RHP Zero located at $s=\dfrac{g_m}{C_f}$

For typical values, for example, in the μA741 of $g_m=6.8mA/V$ and $C_f=30pF$,

$$s=\frac{g_m}{C_f}=\frac{6.8\times10^{-3}}{30\times10^{-12}}\approx 36MHz$$

- The RHP zero is usually not a problem in bipolar op amps because it is at a much higher frequency than the dominant pole, and can be neglected!
- In a CMOS op amp, where transistor transconductances can be much lower, the right-half plane zero frequency can be quite close to the unity-gain frequency of the op amp.

3-dB Frequency
Poles and Zeros Can Be Easily Determined

Low-Frequency Response	High-Frequency Response
$A(s)\cong A_M F_L(s)$	$A(s)\cong A_M F_H(s)$
$F_L(s)=\dfrac{(s+\omega_{Z1})(s+\omega_{Z2})\cdots(s+\omega_{Zn_L})}{(s+\omega_{P1})(s+\omega_{P2})\cdots(s+\omega_{Pn_L})}$	$F_H(s)=\dfrac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn_H})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn_H})}$
If $\omega_{P1}>>\omega_{P2},\ \omega_{P3},\ \ldots,\ \omega_{Z1},\ \omega_{Z2},\ldots$ then for frequencies near the midband:	If $\omega_{P1}>>\omega_{P2},\ \omega_{P3},\ \ldots,\ \omega_{Z1},\ \omega_{Z2},\ldots$ then for frequencies near the midband:
$F_L(s)\cong\dfrac{s}{s+\omega_{P1}}$ (Dominant pole)	$F_H(s)\cong\dfrac{1}{1+s/\omega_{P1}}$ (Dominant pole)
and, $\omega_L\cong\omega_{P1}$.	and, $\omega_H\cong\omega_{P1}$.
Otherwise,	Otherwise,
$\omega_L\cong\sqrt{\omega_{P1}^2+\omega_{P2}^2+\cdots-2(\omega_{Z1}^2+\omega_{Z2}^2+\cdots)}$	$\omega_H=1\Big/\sqrt{\dfrac{1}{\omega_{P1}^2}+\dfrac{1}{\omega_{P2}^2}+\cdots-2\left(\dfrac{1}{\omega_{Z1}^2}+\dfrac{1}{\omega_{Z2}^2}+\cdots\right)}$

3-dB Frequency
Poles and Zeros Cannot Be Easily Determined

Low-Frequency Response	High-Frequency Response
$F_L(s)=\dfrac{s^{nL}+d_1 s^{nL-1}+\cdots}{s^{nL}+e_1 s^{nL-1}+\cdots}$	$F_H(s)=\dfrac{1+a_1 s+a_2 s^2+\cdots}{1+b_1 s+b_2 s^2+\cdots}$
$e_1=\omega_{P1}+\omega_{P2}+\cdots+\omega_{PnL}$	$b_1=\dfrac{1}{\omega_{P1}}+\dfrac{1}{\omega_{P2}}+\cdots+\dfrac{1}{\omega_{PnH}}$
$e_1=\sum_{i=1}^{nL}\dfrac{1}{C_i R_{is}}$	$b_1=\sum_{i=1}^{nH} C_i R_{io}$
If a dominant pole exists (say, P_1), then	If a dominant pole exists (say, P_1), then
$e_1\cong\omega_{P1}$ and $\omega_L\cong\omega_{P1}$	$b_1\cong\dfrac{1}{\omega_{P1}}$ and $\omega_H\cong\omega_{P1}$
Thus,	Thus,
$\omega_L\cong\sum_{i=1}^{nL}\dfrac{1}{C_i R_{is}}$	$\omega_H\cong 1\Big/\sum_{i=1}^{nL} C_i R_{io}$

Dominant Pole

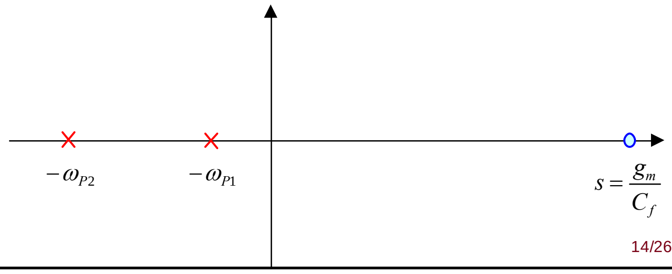
$$\frac{V_o}{I_i}=\frac{(sC_f-g_m)R_1R_2}{1+s[C_1R_1+C_2R_2+C_f(g_mR_1R_2+R_1+R_2)]+s^2[C_1C_2+C_f(C_1+C_2)]R_1R_2}$$

If the RHP Zero be neglected,

$$\frac{V_o}{I_i}\approx\frac{-g_mR_1R_2}{1+s[C_1R_1+C_2R_2+C_f(g_mR_1R_2+R_1+R_2)]+s^2[C_1C_2+C_f(C_1+C_2)]R_1R_2}$$

If the two poles are two separated real poles,

$$\frac{V_o}{I_i}=\frac{-g_mR_1R_2}{\left(1+\dfrac{s}{\omega_{p1}}\right)\left(1+\dfrac{s}{\omega_{p1}}\right)}$$



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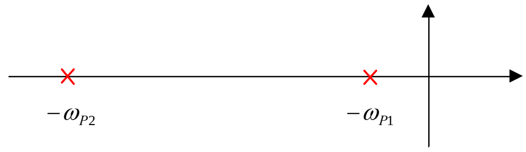
Dominant Pole

If $\omega_{p1} << \omega_{p2}$, then the ω_{p1} can be regarded as the **Dominant Pole**.



The denominator polynominal D(s) can be written as:

$$D(s) = \left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p1}}\right) = 1 + s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}} \approx 1 + s\frac{1}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$\frac{V_o}{I_i} \approx \frac{-g_m R_1 R_2}{1+s[C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_f(C_1+C_2)]R_1R_2}$$

$$\Rightarrow \quad \omega_{p1} = \frac{1}{C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)} \approx \frac{1}{C_f g_m R_1 R_2}$$

Dominant Pole

$$D(s) = 1 + s\frac{1}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$= 1 + s[C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_f(C_1+C_2)]R_1R_2$$

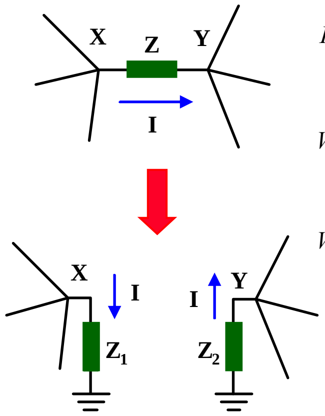
$$\omega_{p1}\omega_{p2} = \frac{1}{[C_1C_2 + C_f(C_1+C_2)]R_1R_2}$$

$$\omega_{p1} \approx \frac{1}{C_f g_m R_1 R_2} \Rightarrow \omega_{p1} \approx \frac{1}{R_1 C} \xrightarrow{\text{Miller Effect}} C = g_m R_2 C_f = A_V C_f$$

$$\omega_{p2} = \frac{g_m C_f}{C_1C_2 + C_f(C_1+C_2)}$$

Miller Theorem

Miller theorem describes the way to convert a floating load into two grounded loads, in such way that the voltages and currents are remained unchanged.



$$I = \frac{V_X - V_Y}{Z}, \quad A = \frac{V_Y}{V_X}$$

$$V_X = Z_1 I = Z_1 \frac{V_X - V_Y}{Z} \Rightarrow Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} \quad \rightarrow \quad Z_1 = \frac{Z}{1 - A}$$

$$V_Y = Z_2 I = Z_2 \frac{V_Y - V_X}{Z} \Rightarrow Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}} \quad \rightarrow \quad Z_2 = \frac{Z}{1 - \frac{1}{A}}$$

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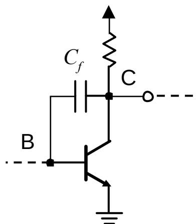
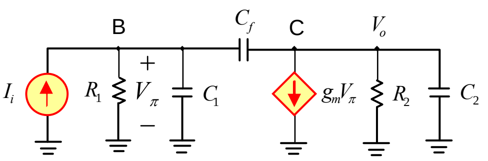
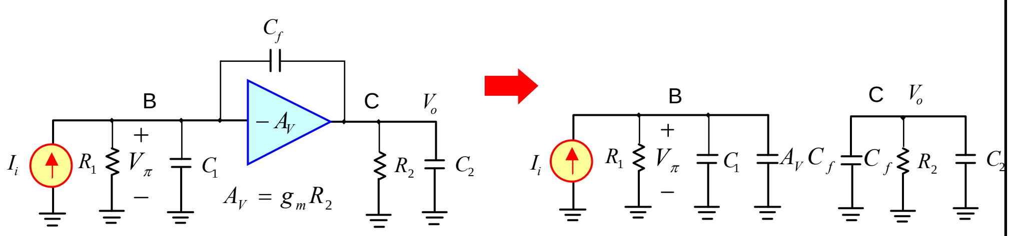
Miller Effect on the Compensation Capacitance



(a) Common-emitter amp.



(b) Equivalent circuit



(c) Equivalent circuit

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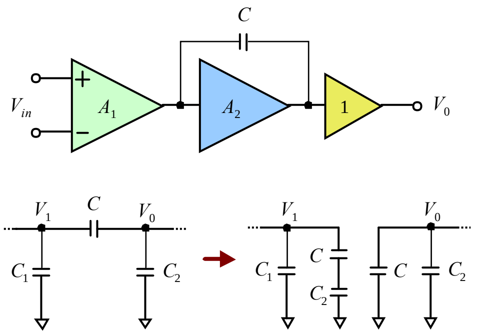
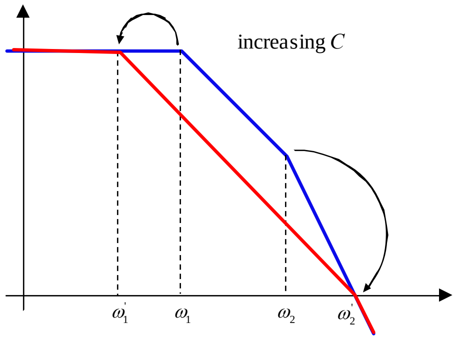
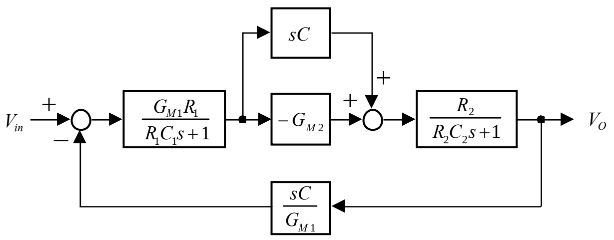
Poles Splitting By Miller Effect





High-frequency model of the effective capacitive loading by the compensation capacitor. When the compensation capacitor C is removed from the circuit at left, the circuit is transformed into the circuit at right so that the capacitive loading on each stage is maintained.

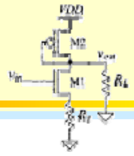
Block Diagram of the OP-AMP Equivalent Circuit



Block diagram of the op-amp equivalent circuit, including the feedforward current through the compensation capacitor. The feedforward term causes a right half-plane zero in the op-amp transfer function.

Kent H. Lundberg, "Internal and External Op-Amp Compensation: A Control-Centric Tutorial," American Control Conference, 2004.

RHP Zero of a Common-Source CMOS Amp



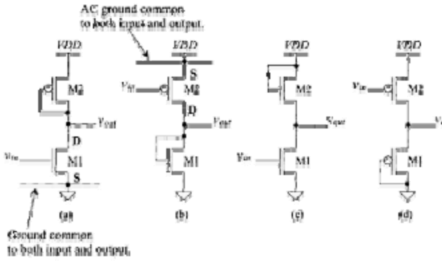


Figure 21.1 Four possible configurations of common-source amplifiers with gate-drain loads.

The Right-Half Plane Zero

It should be noted that when using Miller's theorem a zero is neglected. If we take a look at Fig. 21.5a, we see that at high frequencies C_F shorts the amplifier's output to its input. Our model in Fig. 21.5b doesn't show this shorting. What this means for the amplifier in Fig. 21.1a or its model in Fig. 21.6a is that C_{gd1} shorts the output to the input at high frequencies.

To characterize this high-frequency effect, where we can't use Miller's theorem, consider the circuit seen in Fig. 21.7. In this figure we model the amplifier in Fig. 21.6 (or Fig. 21.1) with a small-signal resistance on the output of the circuit called $R_o = \frac{1}{g_{m2}}||r_{o2}||r_{o1} \approx \frac{1}{g_{m2}}$ and a capacitance on the output, without including C_{gd1}, of $C_o = C_{gs2}$. We don't concern ourselves with the input time constant but rather just look at the output of the circuit. Summing the currents on the output of the model results in

$$\frac{v_{out} - v_{in}}{1/j\omega C_{gd1}} + \frac{v_{out}}{R_o || 1/j\omega C_o} + g_{m1} \cdot v_{in} = 0 \qquad (21.13)$$

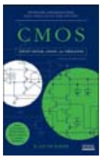
Solving for v_{out}/v_{in} gives

$$\frac{v_{out}}{v_{in}} = -g_{m1}R_o \cdot \frac{1 - j\omega\frac{C_{gd1}}{g_{m1}}}{1 + j\omega(C_{gd1} + C_o)\cdot R_o} \qquad (21.14)$$

The pole should be recognized as what we wrote in Eq. (21.12) if the gain of the amplifier is small ($A_v = -g_{m1}/g_{m2} \approx 0$). In the numerator we see a right-half plane zero at

$$f_z = \frac{g_{m1}}{2\pi C_{gd1}} \qquad (21.15)$$

A zero in the right-half plane (RHP) results in the same magnitude response as a zero in the left-half plane (LHP). However, the phase response is different. A zero in the RHP has the same influence on the phase of a system as a pole in the LHP. Understanding this



CMOS: circuit design, layout, and simulation,
R. Jacob Baker, IEEE Press, 2nd Ed., 2005.

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Limitations Imposed by RHP Poles and Zeros

31.5 Limitations Imposed by Right Half-Plane Poles and Zeros

31.5.1 Introduction

As discussed in Section 31.2, design specifications are often stated in terms of frequency-dependent bounds on the magnitude of closed-loop transfer functions. It has long been known that control system design is more difficult for nonminimum phase or unstable systems. The sensitivity and complementary sensitivity integrals presented in Section 31.4 indicated that nonminimum phase zeros and unstable poles could worsen the individual design tradeoffs. In fact, right half-plane poles and zeros impose additional constraints upon the control system design. This section examines these limitations in detail.

31.5.2 Limitations for Nonminimum Phase Systems

Suppose that the plant possesses zeros in the open right half-plane. Then the internal stability requirement dictates that these zeros also appear, with at least the same multiplicity, in the open-loop transfer function $L(s) = P(s)C(s)$. Let the set of all open right half-plane zeros of $L(s)$ (including any present in the compensator) be denoted by

$$\mathbf{Z} \equiv \{z_i : i = 1, \ldots, N_z\} \qquad (31.44)$$

Defining the Blaschke product (all-pass filter)

$$B_z(s) = \prod_{i=1}^{N_z} \frac{z_i - s}{z_i + s} \qquad (31.45)$$

we can factor the open-loop transfer function into the form

$$L(s) = L_m(s) \prod_{i=1}^{N_z} \frac{z_i - s}{z_i + s} \qquad (31.46)$$

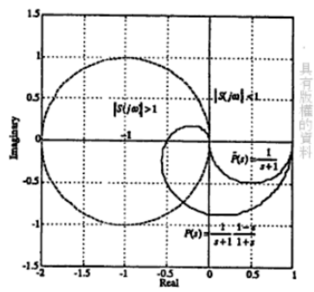


Figure 31.8 Additional phase lag contributed by a nonminimum phase zero.

Assume that the open-loop transfer function can be factored as

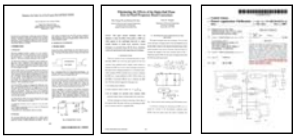
$$L(s) = L_0(s)B_z(s)B_p^{-1}(s)e^{-s\tau} \qquad (31.49)$$

where $\tau \geq 0$ represents a possible time delay, $L_0(s)$ is a proper rational function with no poles or zeros in the open right plane, and $B_z(s)$ is the Blaschke product (Equation 31.45) containing the open right half-plane zeros of the plant plus those of the compensator. The Blaschke product

$$B_p(s) = \prod_{i=1}^{N_p} \frac{p_i - s}{p_i + s} \qquad (31.50)$$

The control handbook, Editor: W. S. Levine

RHP Zero Compensation and Cancellation



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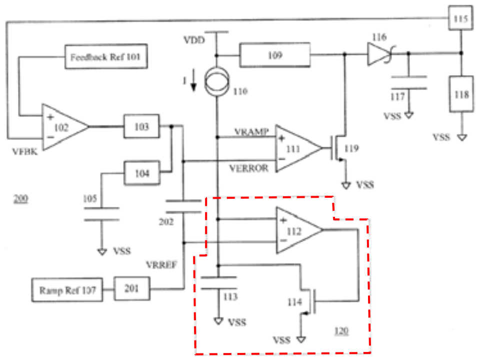
[2] Wei-Chung Wu, R. M. Bass, and J. R. Yeargan, "Eliminating the effects of the right-half plane zero in fixed frequency boost converters," IEEE PESC Con. Rec., 1998.
[3] **Right half-plane zero compensation and cancellation for switching regulators**, Inventor: David W. Ritter, Assignee: Micrel, Incorprated, San Jose, CA., Patten No.: US 2007/0252570 A1, Nov. 1, 2007.

RHP Zero Compensation and Cancellation
US 2007/0252570 A1, Nov. 1, 2007.

Abstract—An improved method of canceling a RHPZ of a switching regulator can include detecting a predetermined error signal provided to a pulse width modulation (PWM) circuit, wherein the predetermined error signal is associated with the RHPZ. Once a RHPZ is detected, a ramp waveform provided to the PWM circuit can be temporarily lengthened, thereby canceling the RHPZ. Notably, temporarily lengthening the ramp waveform can be based on adjusting an RZ*CZ time constant. In one embodiment, the ramp waveform can be lengthened to create a left half-plane zero (LHPZ), which improves stability.



Summary RHP Zero

- Split LC switching converters operating in CCM will have RHP zero due to the lagging inductor effect!
- RHP zero CAN NOT be eliminated with current mode control!
- EHP zero can be eliminated by using variable frequency control.

References: About RHP Zero

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[4] D. M. Sable, B. H. Cho, and R. B. Ridley, "Elimination of the positive zero in fixed frequency boost and flyback converters," *IEEE APEC Proc.*, 1990.
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[6] **Right half-plane zero compensation and cancellation for switching regulators**, Inventor: David W. Ritter, Assignee: Micrel, Incorporated, San Jose, CA., Patten No.: US 2007/0252570 A1, Nov. 1, 2007.