電力電子的基礎理論

Quality Factor - A Measure of Energy Storage Components

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2nd-Order Filter

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The general second-order or ***biquadratic*** filter transfer function is usually expressed in the standard form

$$T(s)=\frac{a_2s^2+a_1s+a_0}{s^2+(\omega_0/Q)s+\omega_0^2}=\frac{a_2s^2+a_1s+a_0}{s^2+2\xi\omega_0 s+\omega_0^2}$$

where ω_0 and Q determine the natural modes (poles) according to

$$p_1,p_2=\omega_0\left[\frac{1}{2Q}\pm j\sqrt{1-\left(1/4Q^2\right)}\right]$$

Definition of the parameters ω_0 and Q of a pair of complex conjugate poles.

Pole Quality Factor $Q=\dfrac{1}{2\xi}$

where ζ is the damping ratio.

Damping Ratio and Pole Quality Factor

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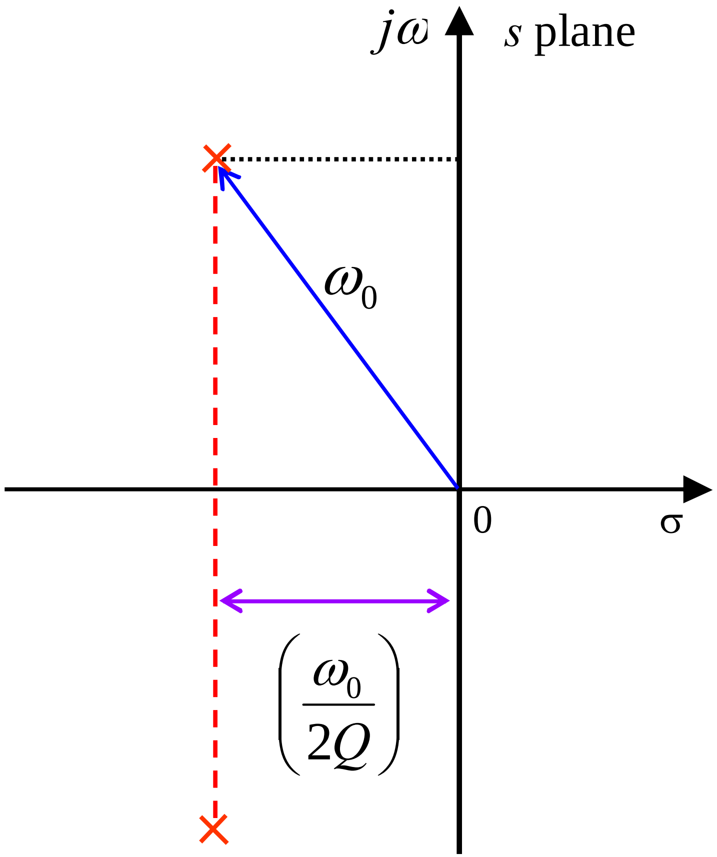

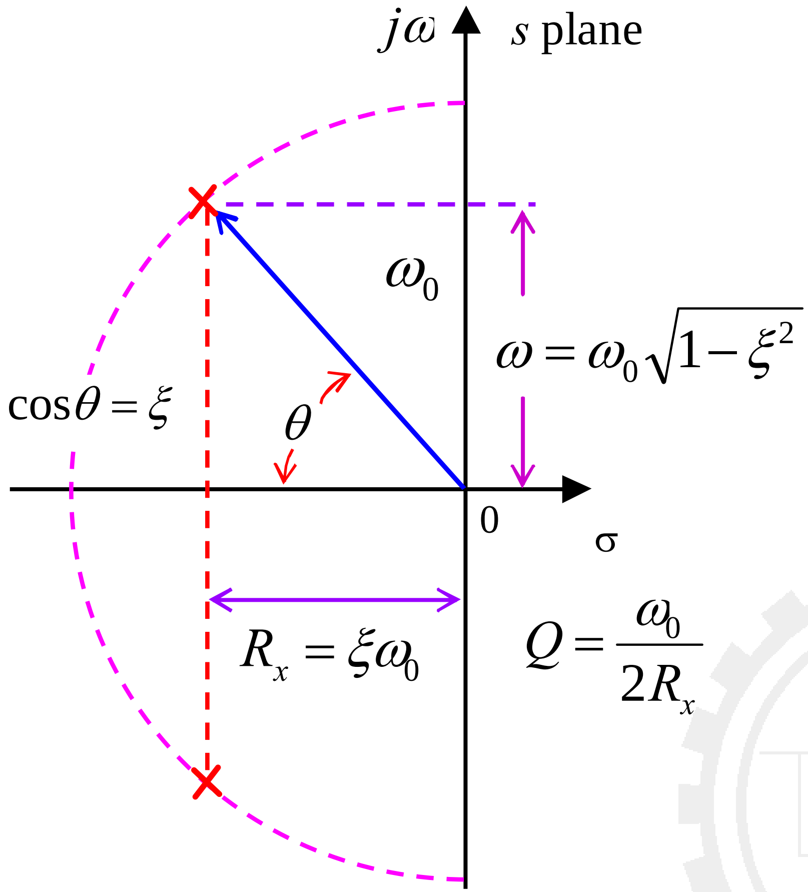

Pole Quality Factor $Q = \frac{1}{2\xi} = \frac{1}{2 \cdot \text{Damping Factor}}$

LC Resonant Tank

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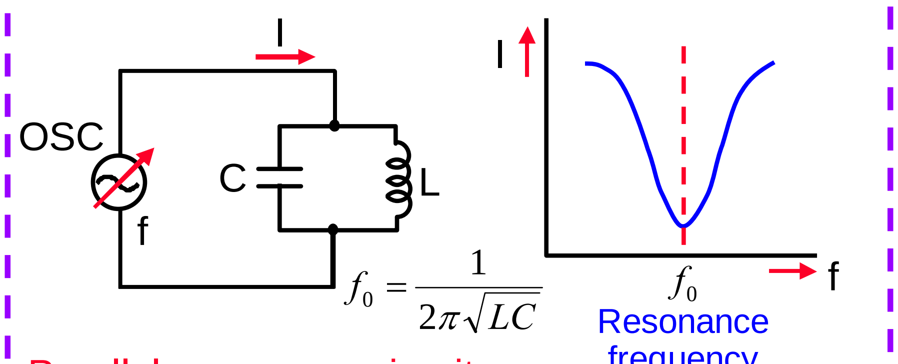
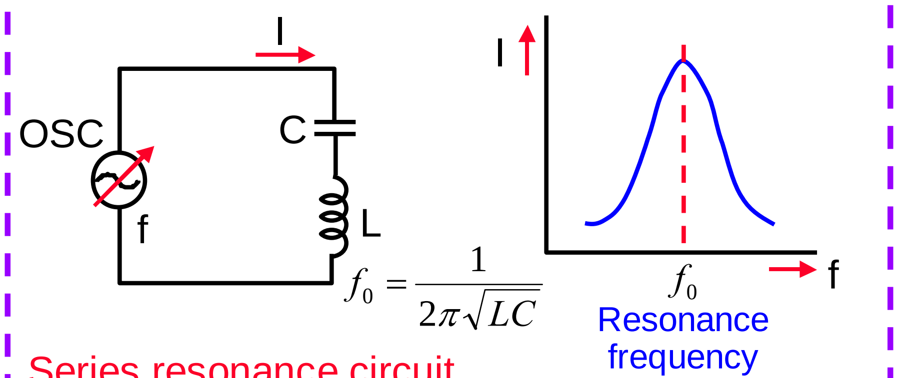


Parallel resonance circuit



Series resonance circuit

When a coil and a capacitor are combined, the resulting circuit has special characteristics.

The impedance (resistance to current flow) of the circuit changes with the frequency of the voltage. Current will flow easily at a given frequency, but has difficulty flowing at another frequency.

The tuning circuit that select a particular radio station utilizes this characteristic.

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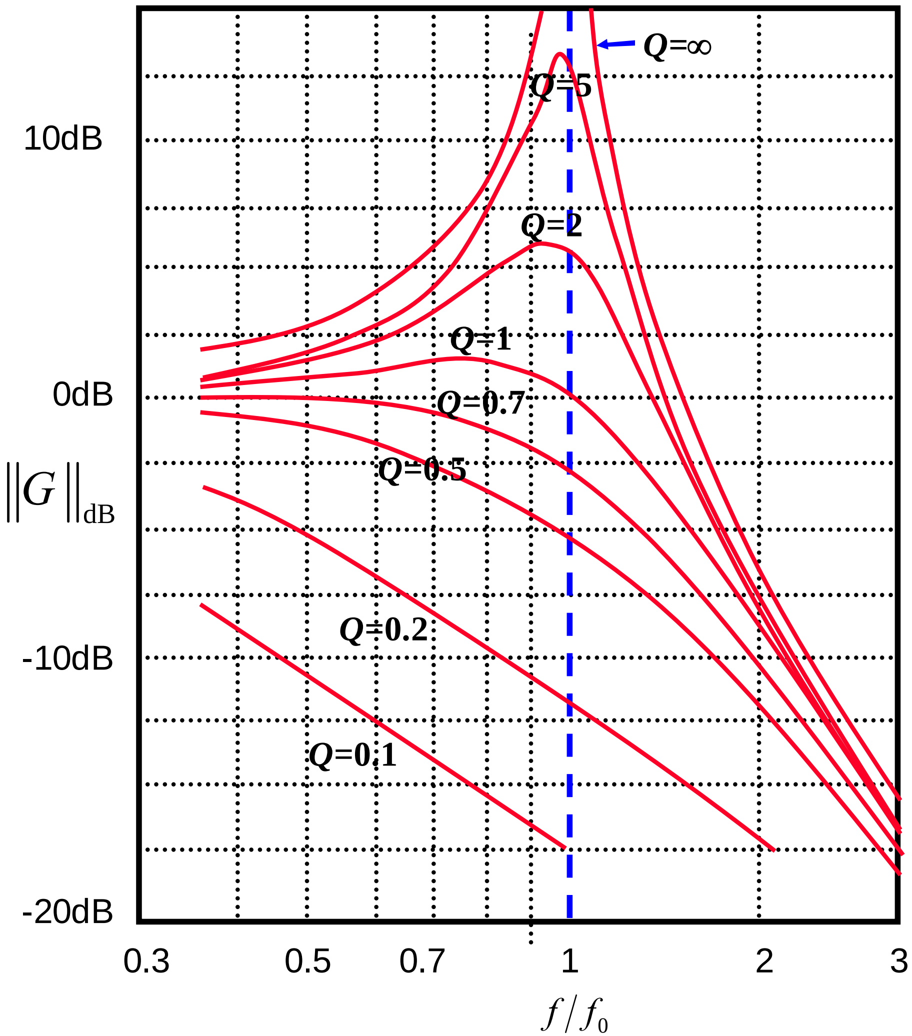
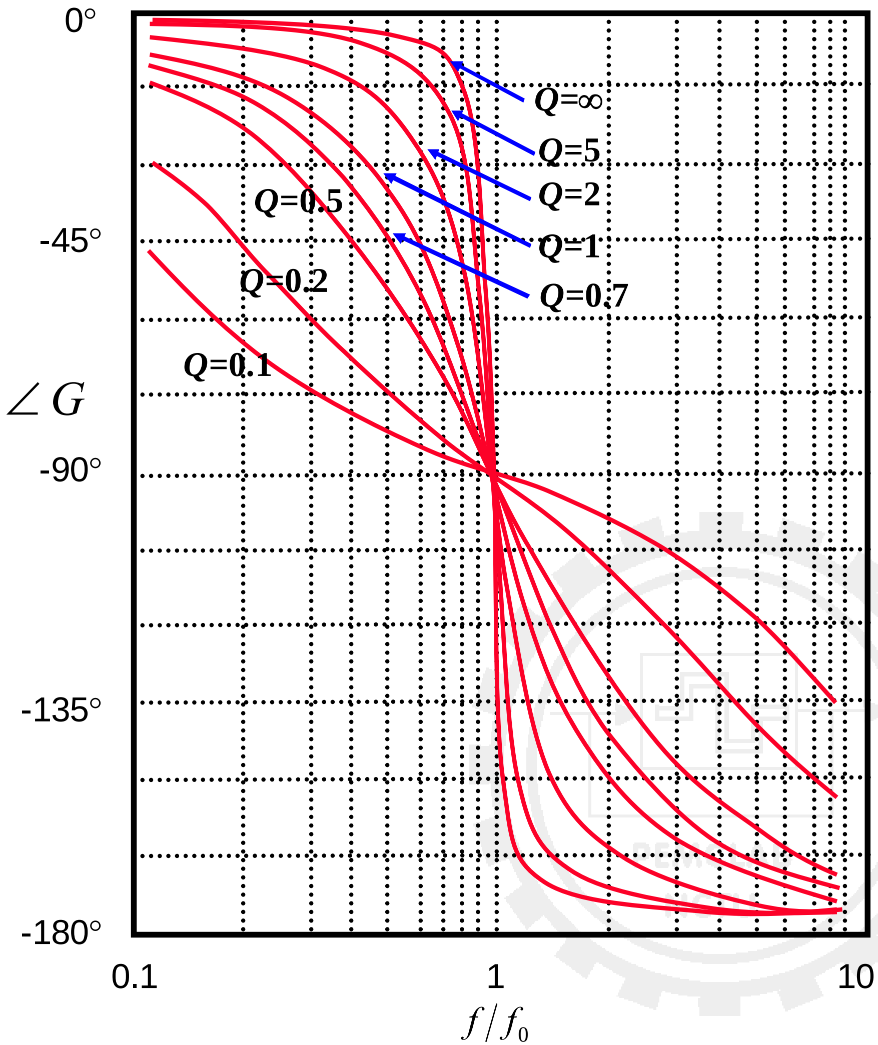
Two-Pole Response: Exact Curves





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Quality Factor (Q)

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An energy analysis of a RLC circuit provides a basic definition of the quality factor (Q) that is used across engineering disciplines, specifically:

$$Q = 2\pi \frac{W_S}{W_D} = 2\pi \frac{Max\ Energy\ Stored\ at\ \omega_0}{Energy\ Dissipated\ per\ Cycle}$$

The quality factor is a measure of the sharpness of the resonance peak; the larger the Q value, the sharper the peak

$$Q = \frac{\omega_0}{BW}$$

where BW=bandwidth

Quality Factor (Q)

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Q-factor is the ratio of its reactance to its resistance.

Q-factor is a measure of the "quality" of a resonant system. Resonant systems respond to frequencies close to the natural frequency much more strongly than they respond to other frequencies.

On a graph of response versus frequency, the bandwidth can be defined as the “full width at half maximum” or FWHM. The Q-factor is defined as the resonant frequency divided by the bandwidth:

$$Q = \frac{\omega_o}{\Delta\omega}$$

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Physical Characteristics of Capacitor

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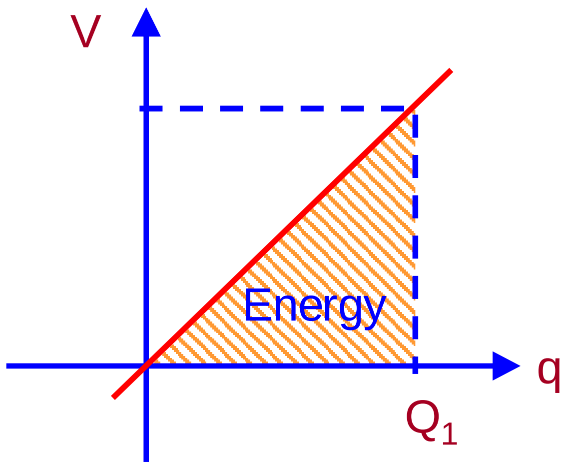
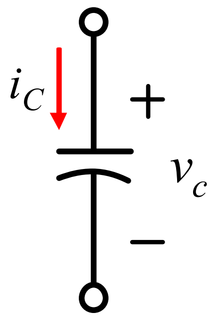
$$i = C\frac{dv}{dt}$$





The current through the capacitor (measure with reference direction) is given by as

$$i(t) = C\frac{dv_c}{dt}$$

$$i(t) = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(t)dt$$

Linear Capacitor: Ideal Capacitor

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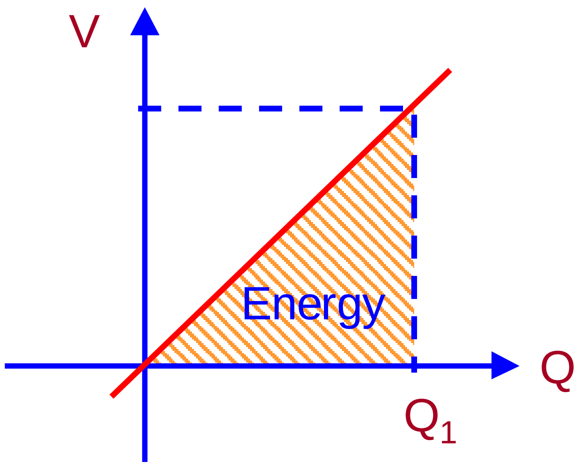
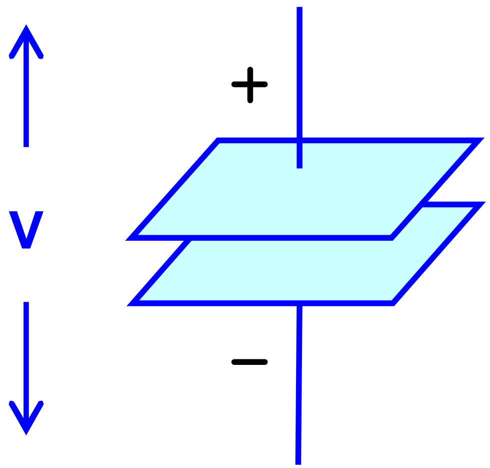
Linear Capacitor:





$$V = \frac{Q}{C} \text{ [volts]}$$

Energy Stored in a Capacitor:

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

$$p(t) = v_c(t)i(t)$$

$$W(t) = \int_0^t p(\tau)d\tau = \int_0^t v_c(\tau)i(\tau)d\tau = \int_{Q_1}^{Q_2} v_c(q)dq$$

$$= \int_{V_1}^{V_2} v_c C dv_c = \frac{C}{2}v_c^2\Big|_{V_1}^{V_2} = \frac{C}{2}(V_2^2 - V_1^2) = \int_{Q_1}^{Q_2}\frac{1}{C}qdq = \frac{1}{2C}(Q_2^2 - Q_1^2)$$

Dielectric Constants

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Dielectric constant (k) gets it's value by comparison of the charge holding ability of a vacuum where k=1. Thus, k is the ratio of the capacitance with a volume of dielectric compared to that of a vacuum dielectric.

$k = \varepsilon_d / \varepsilon_0$

ε_d is the permittivity of the dielectric

ε_0 is the permittivity of free space

Dielectric constants vary with temperature, voltage, and frequency making capacitors messy devices to characterize.

Dielectric strength

Dielectric strength is a property of the dielectric that is usually expressed in volts per mil (V/.001") or volts per centimeter (V/cm). If we exceed the dielectric strength, an electric arc will flash over and often weld the plates of a capacitor together.

Material	Dielectric Constant
Vacuum	1.0000
Air	1.0006
Paraffin paper	3.5
Glass	5 to 10
Mica	3 to 6
Aluminum oxide	7
Tantalum	11
Wood	2.5 to 8
Rubber	2.5 to 35
Glycerine (15°C)	56
Petroleum	2
Pure Water	81
Ceramics	5 to 18000 +

ESR – Equivalent Series Resistance

ESR is the sum of in-phase AC resistance. It includes resistance of the dielectric, plate material, electrolytic solution, and terminal leads at a particular frequency. ESR acts like a resistor in series with a capacitor (thus the name Equivalent Series Resistance). This resister can cause circuits to fail that look just fine on paper and is often the failure mode of capacitors.

To charge the dielectric material current needs to flow down the leads, through the lead plate junction, through the plates themselves - and even through the dielectric material. The dielectric losses can be thought of as friction of aligning dipoles and thus appear as an increase (or a reduction of the rate of decrease -- this increase is what makes the resistance vs. frequency line to go flat.) of measured ESR as frequency increases.

As the dielectric thickness increases so does the ESR. As the plate area increases, the ESR will go down if the plate thickness remains the same.

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Source: http://xtronics.com/reference/esr.htm

Q Factor of a Capacitor

The Q-factor of a capacitor is X_c/R_p

where $X_c=1/\omega C$ and Rp is the equivalent parallel resistance that represents the dielectric and conduction losses.

The Q-factor of a resonant circuit is a measure of the circuit's peak response at the resonant frequency and also its band-width. The greater the Q, the higher the peak response and the narrower the bandwidth.

For a series RLC resonant circuit, $Q=\omega_0 L/R=1/\omega_0 CR$, where ω_0 is the resonant angular frequency, $\omega_0=1/(LC)^{1/2}$. The width of the resonant response curve between half-power point is $\Delta\omega=\omega_0/Q$.

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Q Factor or Quality Factor

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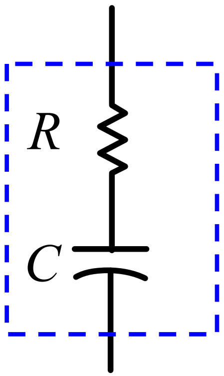
The Q of a capacitor is important in tuned circuits because they are more damped and have a broader tuning point as the Q goes down.

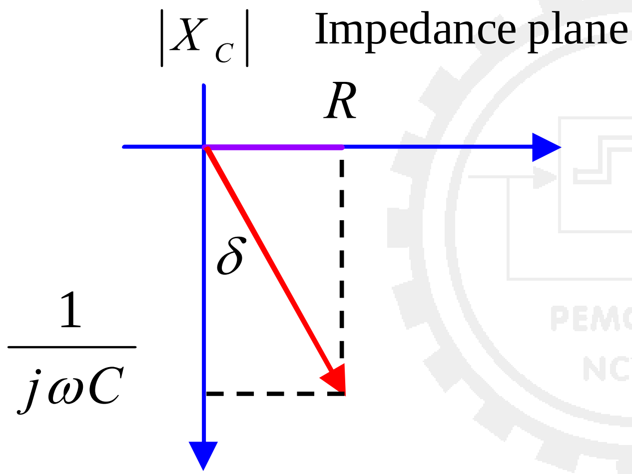
$Q = X_C/R$ where X_C is the capacitive reactance where $X_C = 1/(\omega C)$ and R represents the equivalent series resistance (ESR).

Q is proportional to the inverse of the amount of energy dissipated in the capacitor. Thus, ESR rating of a capacitor is inversely related to its quality.



$$Q = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega RC}$$

$$DP = \tan\delta = \omega CR = \frac{1}{Q}$$



Dissipation Factor (DF)

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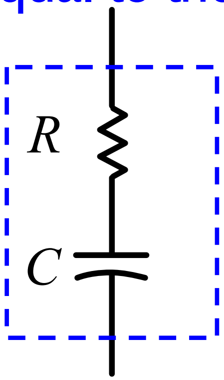
In physics, the dissipation factor (DF) is a measure of loss-rate of power of a mechanical mode, such as an oscillation, in a dissipative system.

For example, electric power is lost in all dielectric materials, usually in the form of heat. DF is expressed as the ratio of the resistive power loss to the capacitive power, and is equal to the tangent of the loss angle.



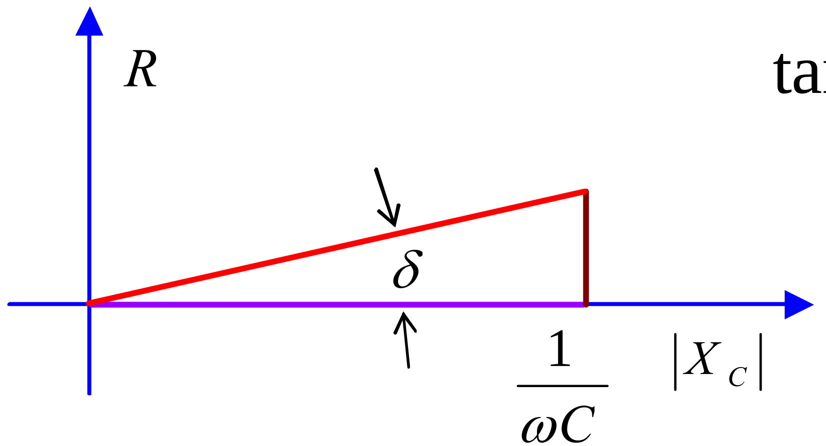
$$DP = \tan\delta = \frac{R}{X_C} = \omega CR = \frac{1}{Q} \quad \Rightarrow \quad R = \frac{\tan\delta}{\omega C}$$

where

δ : loss angle (損失角)

$\tan\delta$: dissipation factor (散逸因數)

Q : quality factor (品質因數)



$$\Rightarrow \quad R\uparrow \;\rightarrow\; \tan\delta\uparrow$$

Dissipation Factor (DF) and Q Factor

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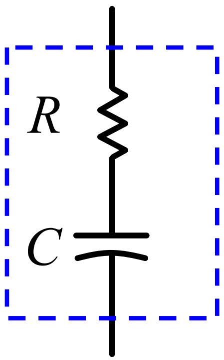
The inverse of Q is the dissipation factor (DF).

$$DF = \frac{1}{Q} = \frac{R}{X_C} = \frac{R}{1/\omega C}$$

The higher the ESR the more losses in the capacitor and the more power we dissipate. If too much energy is dissipated in the capacitor, it heats up to the point that values change (causing drift in operation) or failure of the capacitor.

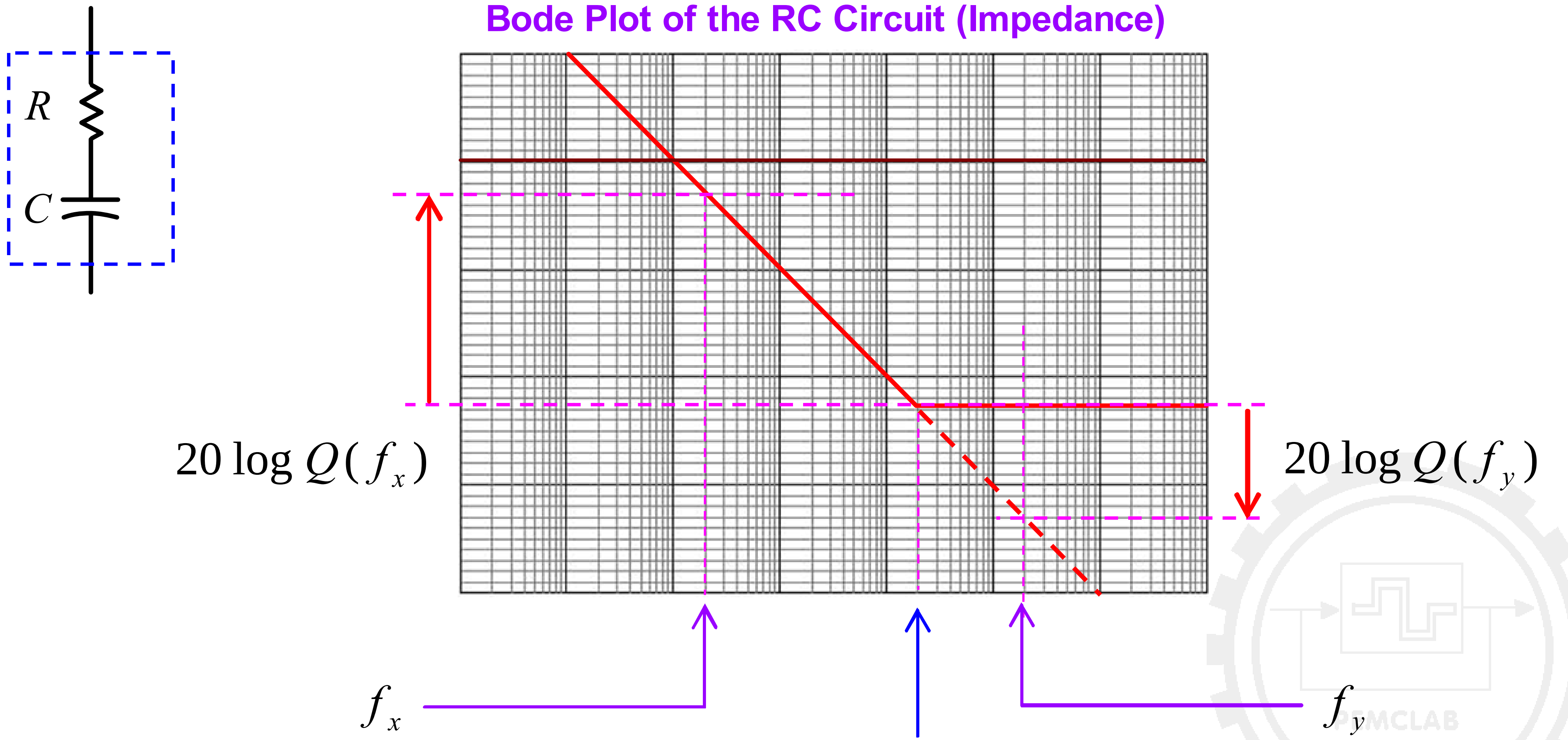


$$Q = \frac{1/\omega C}{R}$$

What is the Frequency Response (Impedance) of the capacitor?

What is the frequency of ω?

Q-Factor and Frequency Response (Impedance)



$$Q = \frac{1/\omega C}{R} = 1$$

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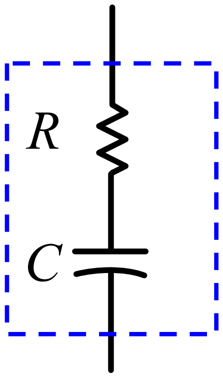
Dissipation Factor (DF) and Q Factor



$$DP = \frac{R}{X_C} = \frac{I_{RMS}^2 R}{I_{RMS}^2 X_C} = \frac{I_{RMS}^2 R}{\frac{1}{2} I_A^2 X_C} = \frac{I_{RMS}^2 R}{\frac{1}{2} I_A^2 \frac{1}{\omega C}} = \frac{I_{RMS}^2 R}{\frac{1}{2} (V_C \omega C)^2 \frac{1}{\omega C}}$$

$$DP = \frac{I_{RMS}^2 R}{\frac{1}{2} (V_C \omega C)^2 \frac{1}{\omega C}} = \frac{I_{RMS}^2 R}{\frac{1}{2} C V_C^2} = \frac{\text{Energy Dissipated in the Resistor}}{\text{Energy Stored in the Capacitor}}$$

$$I_{RMS} = \frac{1}{\sqrt{2}} I_C$$

I_C is the amplitude of the current

$$I_C X_C = I_C \frac{1}{\omega C} = V_C$$

$$I_C = V_C \omega C$$

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Parasitic Elements and Equivalent Circuit of an Inductor

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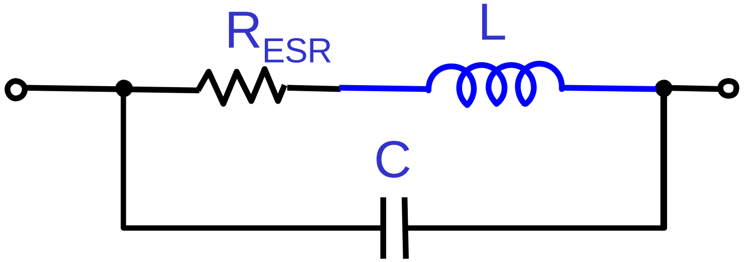
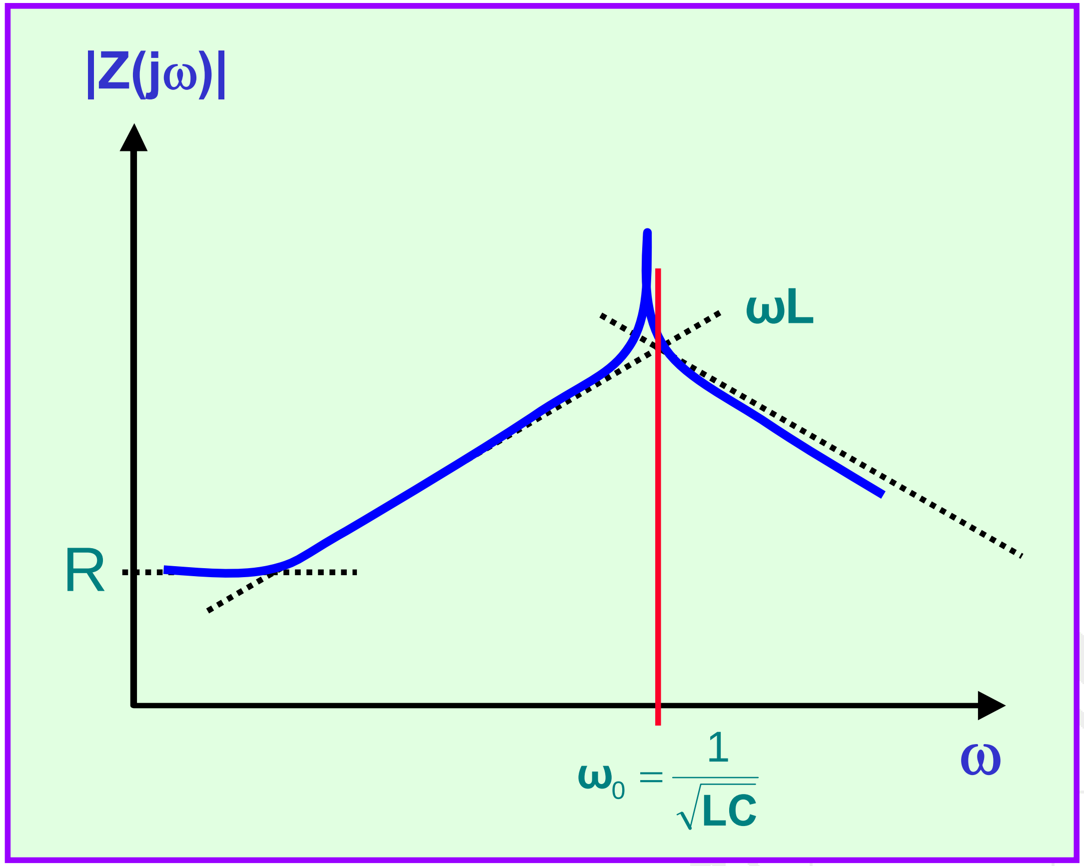


R = series resistance

C = parallel capacitance

$$\mathbf{Z}(\mathbf{j\omega}) = \frac{\mathbf{R} + \mathbf{j\omega L}}{1 + \mathbf{j\omega RC} - \boldsymbol{\omega}^2\mathbf{LC}}$$

Q Factor = $\dfrac{\omega L}{R_{\mathrm{ESR}}}$



The Q-factor of a capacitor is X_L/R_s

where X_L=1/ωC and R_s is the equivalent serial resistance that represents the core and winding losses, ω is the frequency of interest.

3D Embedded High Q-Factor Inductor for Low Frequency Applications

R.F. Drayton & B. Ziaie, E. Davies-Venn, T. Pan, A. Baldi
Electrical & Computer Engineering, University of Minnesota

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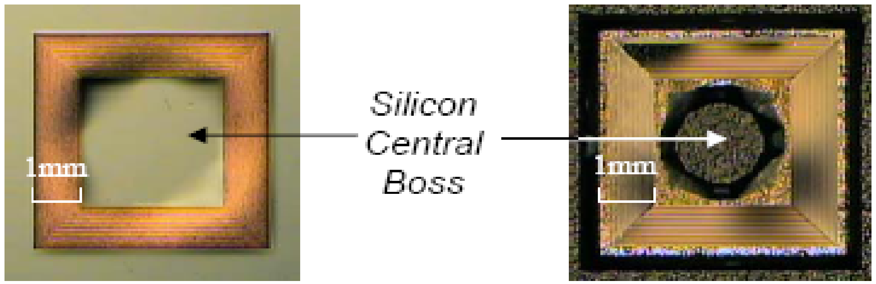
- Project Goal: To develop 3D integrated inductors on CMOS grade silicon substrate for low frequency applications.



(a) (b)

Fig1: Top view (a) and bottom view (b) of 5mm^2 3D embedded inductor.

- Results
  - Large inductance value structures in the low μH range.
  - Quality factors > 60 at ~ 40 MHz. Highest reported in literature at such frequencies.

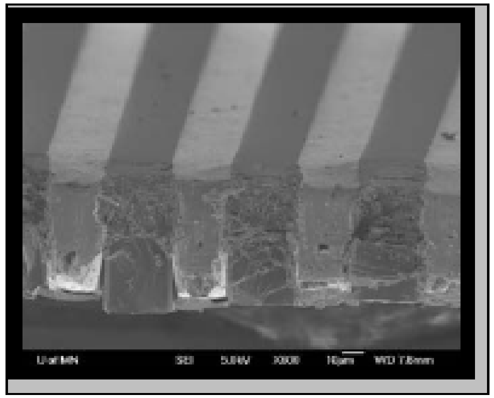


Fig2: SEM cross-section view of 3D embedded inductor in Fig1.

- Publications
  - T. Pan, A. Baldi, E. Davies-Venn, R.F. Drayton, B. Ziaie, "Fabrication and modeling of silicon-embedded high Q inductors", IEEE Int. MEMS Conference, Maastricht, Netherlands, pp. 809-812, Jan 2004.
  - E. Davies-Venn, T. Pan, A. Baldi, R.F. Drayton and B. Ziaie, "Development of Characterization Methods for Micromachined Embedded Test Structures", IEEE SiRF Conference, Atlanta, GA, USA, pp. 318-321, Sept. 2004.

Example: Q-Factor of an LCR Circuit

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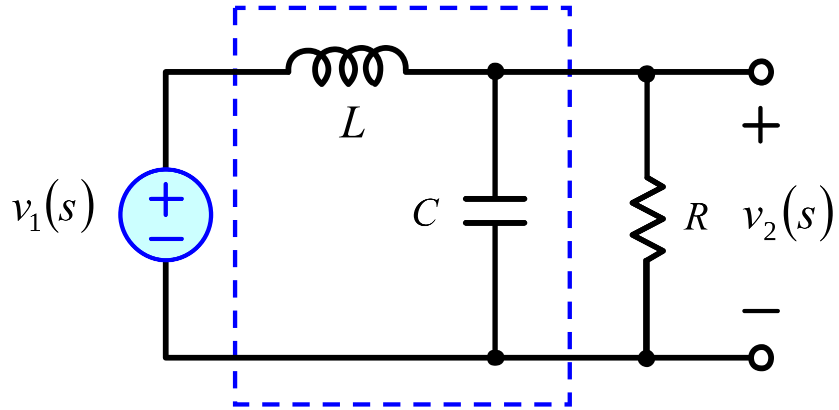
A problem with this procedure is the complexity of the quadratic formula used to find the corner frequencies.

R-L-C network example:



$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

Use quadratic formula to factor denominator.
Corner frequencies are:

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4LC}}{2LC}$$

Factoring the Denominator

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{(L/R)^2 - 4LC}}{2LC}$$

This complicated expression yields little insight into how the corner frequencies ω_1 and ω_2 depend on R, L, and C.

When the corner frequencies are well separated in value, it can be shown that they are given by the much simpler (approximate) expressions

$$\omega_1 \approx \frac{R}{L}, \qquad \omega_2 \approx \frac{1}{RC}$$

ω_1 is then independent of C, and ω_2 is independent of L.

These simpler expressions can be derived via the Low-Q Approximation.

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Derivation of the Low-*Q* Approximation

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Given

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Use quadratic formula to express corner frequencies ω_1 and ω_2 in terms of Q and ω_0 as:

$$\omega_1 = \frac{\omega_0}{Q}\frac{1-\sqrt{1-4Q^2}}{2} \qquad \omega_2 = \frac{\omega_0}{Q}\frac{1+\sqrt{1-4Q^2}}{2}$$

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Corner Frequency ω_2

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$$\omega_2 = \frac{\omega_0}{Q}\frac{1+\sqrt{1-4Q^2}}{2}$$

Can be written in the form

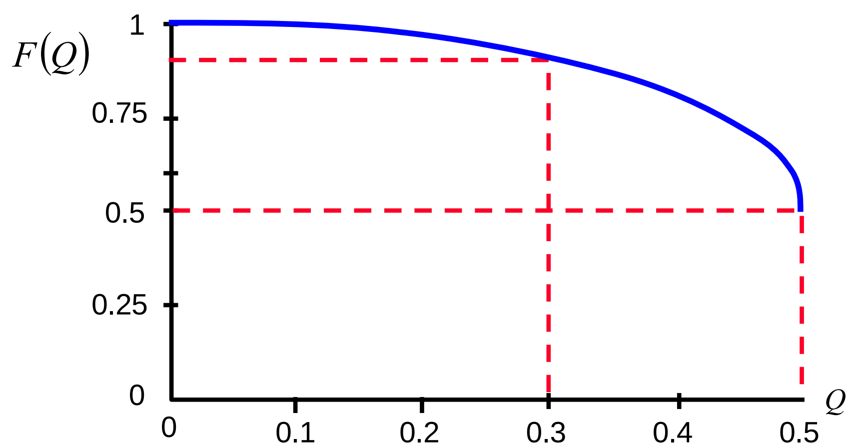
$$\omega_2 = \frac{\omega_0}{Q}F(Q)$$

When

$$F(Q) = \frac{1}{2}\left(1+\sqrt{1-4Q^2}\right)$$

For small Q, tends to 1.
When then obtain

$$\omega_2 \approx \frac{\omega_0}{Q} \quad \text{for } Q << \frac{1}{2}$$



For Q < 0.3, the approximation $F(Q)$ = 1 is within 10% of the exact value.

Corner Frequency ω_1

$$\omega_1 = \frac{\omega_0}{Q}\frac{1-\sqrt{1-4Q^2}}{2}$$

Can be written in the form

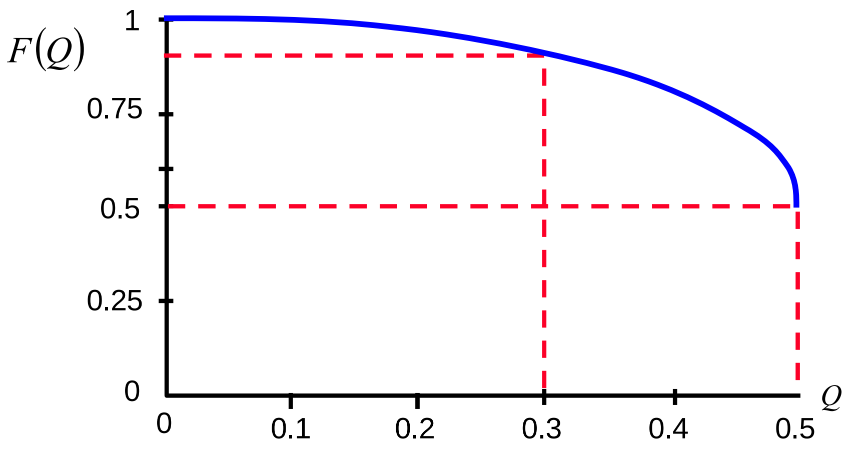
$$\omega_1 = \frac{Q\omega_0}{F(Q)}$$

When

$$F(Q) = \frac{1}{2}\left(1+\sqrt{1-4Q^2}\right)$$

For small Q, $F(Q)$ tends to 1.
When then obtain

$$\omega_1 \approx Q\omega_0 \quad \text{for } Q << \frac{1}{2}$$



For Q < 0.3, the approximation $F(Q)$ = 1 is within 10% of the exact value.

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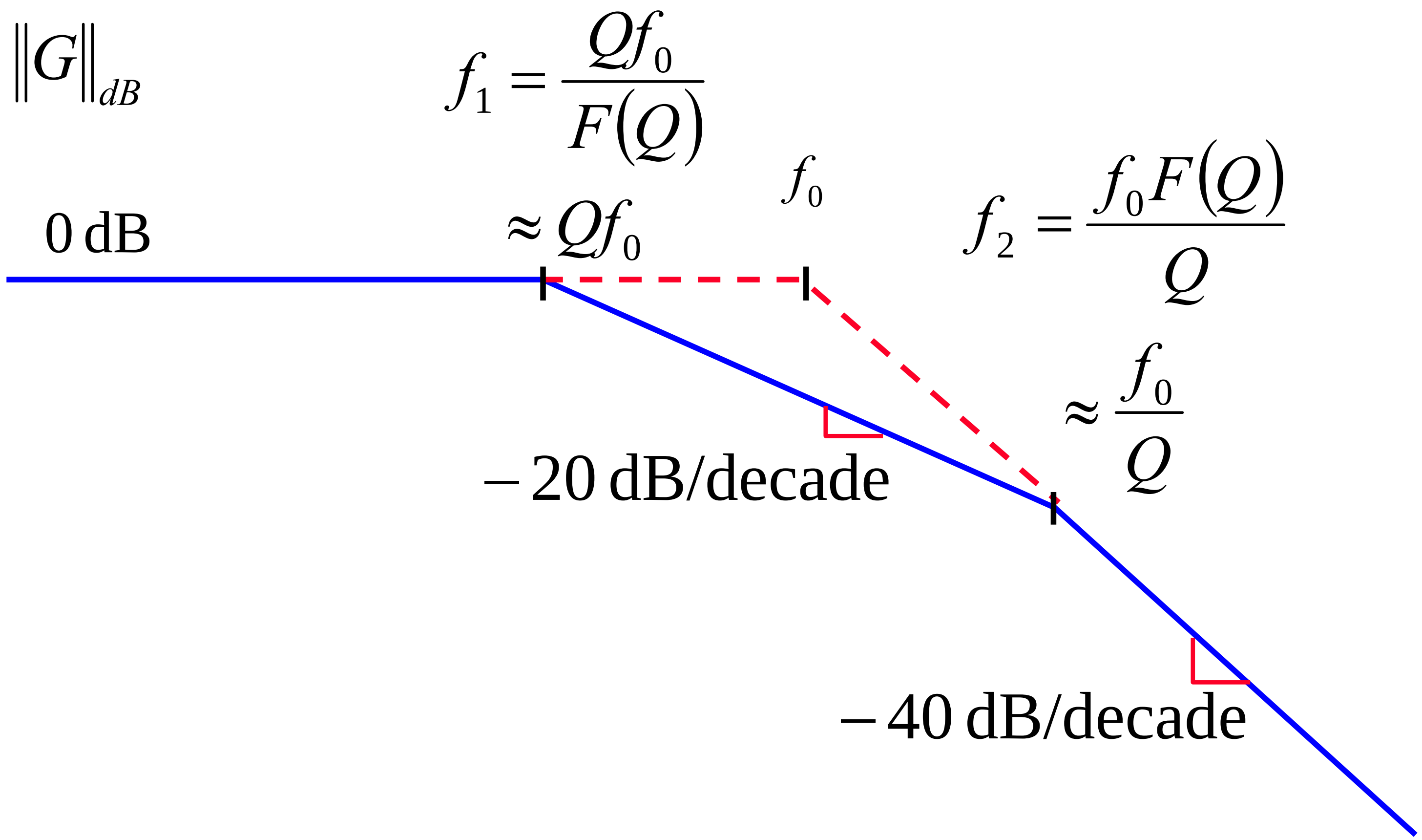
The Low-Q Approximation



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R-L-C Example

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For the previous example:

$$G(s)=\frac{v_2(2)}{v_1(2)}=\frac{1}{1+s\frac{L}{R}+s^2LC}$$

$$f_0=\frac{\omega_0}{2\pi}=\frac{1}{2\pi\sqrt{LC}}$$

$$Q=R\sqrt{\frac{C}{L}}=\frac{R}{\sqrt{\frac{L}{C}}}$$

Use of the low-Approximation leads to

$$\omega_1\approx Q\omega_0=R\sqrt{\frac{C}{L}}\frac{1}{\sqrt{LC}}=\frac{R}{L}$$

$$\omega_2\approx\frac{\omega_0}{Q}=\frac{1}{\sqrt{LC}}\frac{1}{R\sqrt{\frac{C}{L}}}=\frac{1}{RC}$$

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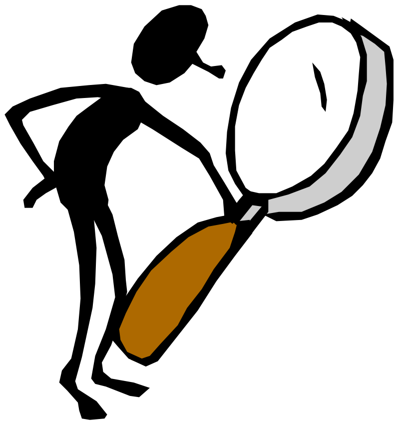
Smart Power Processing for Energy Saving

Thank you for your attention!

Knowledge, Innovation, and Education



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